A Behavioral Explanation for the Puzzling Persistence of the Aggregate Real Exchange Rate

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The Law-of-One-Price and Purchasing Power Parity

- LOP

$$\frac{S_t P^*_{\text{beer},t}}{P_{\text{beer},t}} = 1$$

where S_t is the nominal exchange rate, $*$ denotes "foreign"

- PPP

$$\frac{S_t P^*_t}{P_t} = 1$$

where P_t and P^*_t are CPIs of home and foreign countries

The Law-of-One-Price and Purchasing Power Parity

- LOP deviations (or good-level RER)

$$\frac{S_t P^*_{\text{beer},t}}{P_{\text{beer},t}} = q_{\text{beer},t} \neq 1$$

where S_t is the nominal exchange rate, $*$ denotes "foreign"

- PPP deviations (aggregate RER)

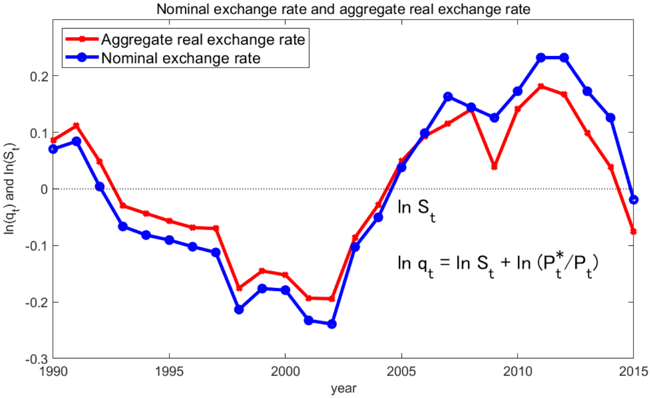
$$\frac{S_t P^*_t}{P_t} = q_t \neq 1$$

where P_t and P^*_t are CPIs of home and foreign countries

- In reality, LOP & PPP do not hold

PPP puzzle 1

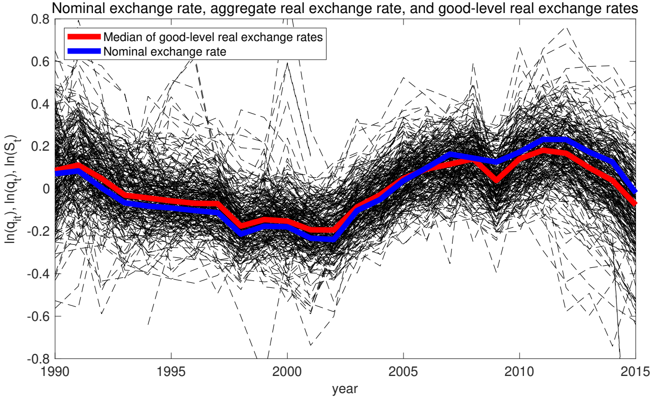
- PPP deviations are extremely persistent



  - too persistent to be explained by reasonable degree of nominal price rigidities (Rogoff 1996)
  - Half-life (HL) = 3-5 years ▸ What's HL?

PPP puzzle 2

- LOP deviations are much less persistent than PPP deviations



- Half-life ($\simeq$ 1.2 year) is lower than that of q_t (3-5 years)
- Imbs, Mumtaz, Ravn and Rey (2005), Crucini and Shintani (2008), Carvalho and Nechio (2011)

Research questions

Q1: Does the behavioral inattention model help solve the PPP puzzle?

- We consider a behavioral model of inattention ("sparse-based model") by Gabaix (2014, 2020)
- Gabaix (2014, 2020) discusses models with attention parameter m:

$$\begin{aligned} m &= 1 \qquad \text{(if agents are fully attentive)} \\ m &< 1 \qquad \text{(if agents are inattentive)} \end{aligned}$$

- We introduce m into the model of LOP deviations used in Crucini, Shintani and Tsuruga (2010a, 2010b, 2013, 2015)

Q2: Do micro price data support the behavioral inattention?

Theoretical finding

Q1: Does the behavioral inattention model help to solve the PPP puzzle?

A1: Yes

- We derive the relationship between LOP deviations and PPP deviations
- If $m = 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + e_{it}$$

(λ: the degree of price stickiness, e: iid shocks)

- If $m < 1$

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_{it}$$

  - **Aggregate RER Dependence** generates persistence of q_t and q_{it}

Theoretical finding

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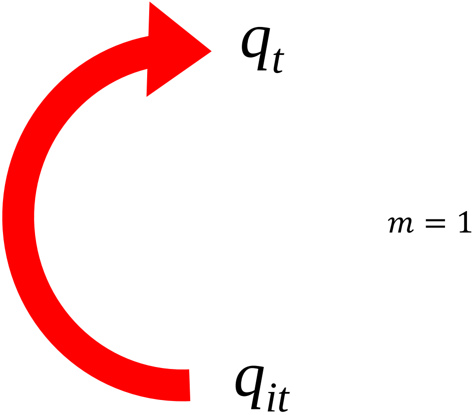
Aggregate RER independence

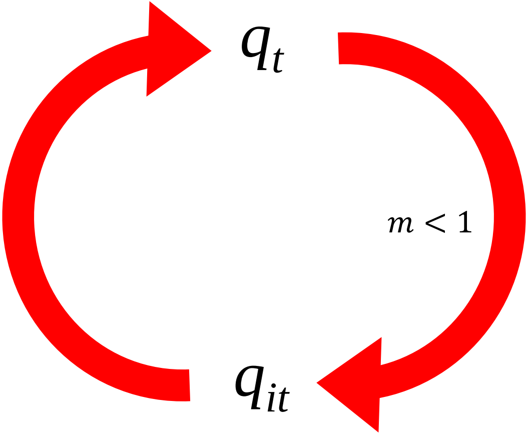
$$\ln q_{it} = \lambda \ln q_{it-1} + e_{it}$$



- q_{it} is independent of aggregate RER if $m = 1$

Aggregate RER dependence

$$\ln q_{it} = \lambda \ln q_{it-1} + (1 - m)(1 - \lambda) \ln q_t + e_{it}$$



- Aggregate RER dependence generates slow aggregate real exchange rate if $m < 1$

Empirical findings

Q2: Do micro price data support the behavioral inattention?

A2: Yes

- We test the model of LOP deviations with behavioral inattention
  - Competing Hypotheses

$$\begin{aligned} H_0 &: \text{Agg. RER independence } (m = 1) \\ H_1 &: \text{Agg. RER dependence } (m < 1) \end{aligned}$$

  - H_0: $m = 1$ is strongly rejected by the data
- Our estimates of m are $m = 0.11 - 0.25$
- Under the estimated m, the model explains the PPP puzzle

The persistence of the PPP and LOP deviations

- With $m = 1$ (**full attention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate RERs (PPP deviations)			
Half-life	**0.6**		**2.4-4.9**
Good-level RERs (LOP deviations)			
Half-life	0.6		1.2-1.6

- Note: The unit of the HL is year

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The persistence of the PPP and LOP deviations

- With $m = 0.1$ (**behavioral inattention**)

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate RERs (PPP deviations)			
Half-life	0.6	**2.5-3.7**	2.4-4.9
Good-level RERs (LOP deviations)			
Half-life	0.6	**1.0-1.2**	1.2-1.6

- Note: The unit of the HL is year

A simple model of behavioral inattention

- LOP deviations are the ratio of individual prices

$$q_{it} = \frac{S_t P^*_{it}}{P_{it}}$$

- Individual prices P_{it} are important
- Focus on the firms' pricing under monopolistic competition
  - in a static setting, for simplicity

A simple model of behavioral inattention

- 2 steps
  1. Firm's profit function under full attention (log-approximated to the second order)

$$\max_{\hat{p}_i} \pi(\hat{p}_i, \hat{w}) = \max_{\hat{p}_i} \left[-\frac{r}{2} (\hat{p}_i - \hat{w})^2 \right]$$

$\hat{p}_i$: firm i's actual (relative) price, $\hat{w}$: the optimal price (e.g. real wages in terms of log-deviations), r: constant

- The fully attentive firms' optimal price

$$\hat{p}_i = \hat{w}$$

  2. Behavioral firms' replace $\pi(\hat{p}_i, \hat{w})$ with "attention augmented profit function"

$$\max_{\hat{p}_i} \tilde{\pi}(\hat{p}_i, \hat{w}, m) = \max_{\hat{p}_i} \left[-\frac{r}{2} (\hat{p}_i - m\hat{w})^2 \right]$$

- In a special case with $m = 1$, $\tilde{\pi}(\hat{p}_i, \hat{w}, 1) = \pi(\hat{p}_i, \hat{w})$
- The action becomes

$$\hat{p}_i(m) = m\hat{w}$$

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     - The action becomes

       $$\hat{p}_i(m) = m\hat{w}$$

Choice of attention (1)

- How do firms determine m?
- Before setting $p_i(m)$, a firm optimally chooses the degree of attention

  $$\max_{m \in [0,1]} \mathbb{E}\left[\pi(\hat{p}_i(m), \hat{w}) - \mathcal{C}(m)\right]$$

  where $\mathcal{C}(m)$ is a psychic cost of paying attention
- Trade-off
  - Benefit: correction of distorted action
  - Cost: paying attention

Choice of attention (2): Example

- Let $\mathcal{C}(m)$ is quadratic in m:

$$\mathcal{C}(m) = \frac{\kappa}{2} m^2$$

- Using $\hat{p} = m\hat{x}$, the objective function can be written as

$$\max_{m \in [0,1]} \mathbb{E}\left[-\frac{r}{2}(\hat{p}_i(m) - \hat{w})^2 - \mathcal{C}(m) \right]$$

$$= \max_{m \in [0,1]} -\frac{r\sigma_w^2}{2}(1-m)^2 - \frac{\kappa}{2} m^2$$

- FOC implies

$$m = \frac{r\sigma_w^2}{r\sigma_w^2 + \kappa}$$

  - Lower κ or higher r leads to more attention
  - More volatile real wage leads to more attention

- Rational Inattention

Behavioral pricing: Interpretation

- Firms maximize real profits augmented with attention

$$\pi(\hat{p}_i, \hat{w}) = -\frac{r}{2}(\hat{p}_i - m\hat{w})^2$$

- Recall $\hat{p}_i$ and $\hat{w}_i$ is log real prices

$$\hat{p}_i = \ln P_i - \ln P \qquad \hat{w} = \ln W - \ln P$$

- Optimal pricing under behavioral inattention implies

$$\hat{p}_i(m) = m\hat{w}$$

- In nominal terms,

$$\ln P_i = (1 - m)\ln P + m \ln W$$

- $\ln P$: the average prices (or aggregate prices)

Behavioral pricing: Interpretation

- Firms' prices deviate from the average price toward nominal wages

$$\ln P_i = (1 - m) \ln P + m \ln W$$

  - the average price $\ln P$ serves as a default value that spontaneously comes to mind without thinking
  - Gabaix (2019)
- With $0 < m < 1$
  - firms start from the default value and make partial adjustment
  - similar to the idea of psychology of "anchoring and adjustment" (Tversky and Kahneman 1974)

Overview of the model

- Follows Kehoe and Midrigan (2007), Crucini, Shintani and Tsuruga (2010, 2013)
- Households
  - $U(c_t, n_t) = \ln c_t - \chi n_t$ max. problem
- Firms
  - Set prices in monopolistically competitive market (Home and Foreign, local currency pricing) CES
  - Calvo pricing with its parameter λ
  - Use technology: $y_{it}(z) = a_{it} n_{it}(z)$
  - Must pay trade cost to send goods from a country to the other resources
  - We introduce Gabaix's behavioral inattention to price setting
- Governments
  - Control money supply

Fully attentive firms' pricing: step 1

- Home firm's pricing under **full attention**

$$\hat{p}_{Hit} = (1-\lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\quad \widehat{mc}_{Hit+k})$$

$$\hat{p}^*_{Hit} = (1-\lambda\delta)\mathbb{E}_t \sum_{k=0}^{\infty} (\lambda\delta)^k (\quad \widehat{mc}^*_{Hit+k})$$

where all variables are the log-deviations, p_{Hit}: relative price of good i, a_{it}: productivity, δ: discount factor,

- $\hat{p}^*_{Fit}$ and $\hat{p}_{Fit}$ are analogously derived

Behavioral firms' pricing: step 2

- Home firm's pricing under **behavioral inattention**

$$\hat{p}_{Hit}(m_H) = (1-\lambda\delta)\mathbb{E}_t\sum_{k=0}^{\infty}(\lambda\delta)^k(m_H\widehat{mc}_{Hit+k})$$

$$\hat{p}^*_{Hit}(m^*_H) = (1-\lambda\delta)\mathbb{E}_t\sum_{k=0}^{\infty}(\lambda\delta)^k(m^*_H\widehat{mc}^*_{Hit+k})$$

- $\hat{p}^*_{Fit}(m^*_F)$ and $\hat{p}_{Fit}(m_F)$ are analogously derived
- The optimal (relative) prices are insensitive to the aggregate shocks price index

Proposition 1

Under the preferences given by $U(c,n) = \ln c - \chi n$, *the CIA constraints, the stochastic processes of money supply, the stochastic processes of the labor productivity, and the Calvo pricing with the degree of price stickiness* $\lambda \in (0,1)$, *the stochastic process of the good-level real exchange rate is given by:*

$$\ln q_{it} = \lambda \ln q_{it-1} + (1-m)(1-\lambda)\ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

where

- $m \in (0,1]$: the degree of attention, $m = \omega m_H + (1-\omega) m_F$
- ψ: param. for real frictions (> 0 with trade cost)
- ε_t^n: nominal shock, $\varepsilon_t^n = \Delta \ln S_t \sim i.i.d.(0, \sigma_n^2)$
- ε_{it}^r: real shock, $\varepsilon_{it}^r = (\varepsilon_{it} - \varepsilon_{it}^*) \sim i.i.d.(0, \sigma_r^2)$

Takeaway: Dependence of the good-level RER $\ln q_{it}$ on the aggregate RER $\ln q_t$

Test of aggregate RER independence

- We define modified LOP deviations $\widetilde{q}_{it}$ and PPP deviations $\widetilde{q}_t$

$$\underbrace{\ln q_{it} - \lambda \ln q_{it-1} - \lambda \varepsilon_t^n}_{\ln \widetilde{q}_{it}} = (1-m) \underbrace{(1-\lambda) \ln q_t}_{\ln \widetilde{q}_t} + \underbrace{\psi \varepsilon_{it}^r}_{u_{it}}$$

where we replace ε_t^n by $\Delta \ln S_t$ and calibrate λ

- Test of aggregate RER independence

$$\ln \widetilde{q}_{it} = \alpha + \beta \ln \widetilde{q}_t + u_{it},$$

- The null hypothesis of aggregate RER independence $(H_0 : \beta = 0)$ is significantly rejected in favor of behavioral inattention

Data

- We use the annual micro price data of US-Canadian city pairs and UK-Euro city pairs
  - The *Worldwide Cost of Living Survey* by Economic Intelligence Unit
  - Our regression has variations in three dimensions

$$\ln q_{ijt}$$

    - 274 US-Canadian goods (i) and 301 UK-European goods (i)
    - 17 US cities and 4 Canadian cities = 68 city pairs (j)
    - or 19 Euro cities and 2 UK cities = 38 city pairs (j)
    - 26 years from 1990 to 2015 (t)

Test of aggregate RER independence (US–Canada)

- Modified RER with common λ

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_t$	0.844***	0.802***	0.812***	0.806***
	(0.030)	(0.028)	(0.029)	(0.029)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.156	0.198	0.188	0.194

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test of aggregate RER independence (UK–Euro)

- Modified RER with common λ

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_t + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_t$	0.856***	0.851***	0.853***	0.868***
	(0.042)	(0.043)	(0.042)	(0.042)
# of Obs.	214,115	214,115	213,064	213,064
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.144	0.149	0.147	0.132

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the median value of the degrees of price stickiness ($\lambda = 0.34$) is used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test of aggregate RER independence (US–Canada)

- Modified RER with good-specific λ_i

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_{it}$	0.894***	0.862***	0.883***	0.880***
	(0.029)	(0.028)	(0.032)	(0.033)
# of Obs.	389,500	389,500	389,500	389,500
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.106	0.138	0.117	0.120

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test for behavioral inattention (UK–Euro)

- Modified RER with good-specific λ_i

$$\ln \widetilde{q}_{ijt} = \alpha_{ij} + \beta \ln \widetilde{q}_{it} + \gamma' X_{ijt} + u_{ijt}$$

	(1)	(2)	(3)	(4)
$\ln \widetilde{q}_{it}$	0.866***	0.834***	0.864***	0.840***
	(0.047)	(0.049)	(0.048)	(0.049)
# of Obs.	171,606	171,606	170,750	170,750
city-pairs FE	N	Y	N	Y
Control for productivity	N	N	Y	Y
$\hat{m}$	0.134	0.166	0.136	0.160

Note: *10% significance level, **5% significance level, ***1% significance level. For λ, the good-specific values of the degrees of price stickiness (λ_i) are used. All specifications include the fixed effect at the good level. The standard errors are clustered by goods.

Test of aggregate RER independence: Summary

- The null hypothesis of $\beta = 0$ is significantly rejected and robust to various specifications
- The estimated degree of inattention ranges between 0.11–0.25
  - Baseline estimate $m = 0.106$ for US-Canada
  - Baseline estimate $m = 0.134$ for UK-Euro
- What are the implications for PPP puzzle?

Propositions 2 & 3

Under the same assumptions in Proposition 1,

$$HL_q > HL_{q|m=1} \text{ (PPP puzzle 1)}$$

$$HL_q > HL_{qi} \text{ (PPP puzzle 2)}$$

provided $m \in (0,1)$, $\lambda \in (0,1)$, $\tau \in (0,\infty)$, *and* $\sigma_r/\sigma_n \in (0,\infty)$

- Eq Prop 2
- Eq Prop 3

The persistence of the PPP and LOP deviations

- For US-Canada with $m = 0.106$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate RERs (PPP deviations)				
Half-life	0.6	3.7	[2.5, 7.6]	4.9
Good-level RERs (LOP deviations)				
Half-life	0.6	1.2	[1.0, 2.1]	1.6

- Note: The unit of the HL is year

The persistence of the PPP and LOP deviations

- For UK-Euro with $m = 0.134$

	Model ($m = 1$)	Model ($m < 1$)	95% CI	Data
Aggregate RERs (PPP deviations)				
Half-life	0.6	2.5	[1.7, 4.9]	2.4
Good-level RERs (LOP deviations)				
Half-life	0.6	1.0	[0.8, 1.5]	1.2

- Note: The unit of the HL is year Taylor rule

Conclusion

- Two puzzles on PPP and LOP deviations

  1. The persistence of the aggregate RER is too high to be explained by reasonable degree of nominal price rigidities

  2. The good-level RER is less persistent than the aggregate RER

- The behavioral model by Gabaix (2014, 2020) could explain these puzzles

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate RERs (PPP deviations)			
Half-life	0.6	2.5-3.7	2.4-4.9
Good-level RERs (LOP deviations)			
Half-life	0.6	1.0-1.2	1.2-1.6

The way forward

1. Roundabout production

   - It may also generate the aggregate RER dependence

     $$y_{it}(z) = a_{it} n_{it}(z)^{1-r} \Gamma_{it}(z)^r$$

     where r is the degree of roundabout production
     $\Gamma_{it}(z)$ is the intermediate input demand

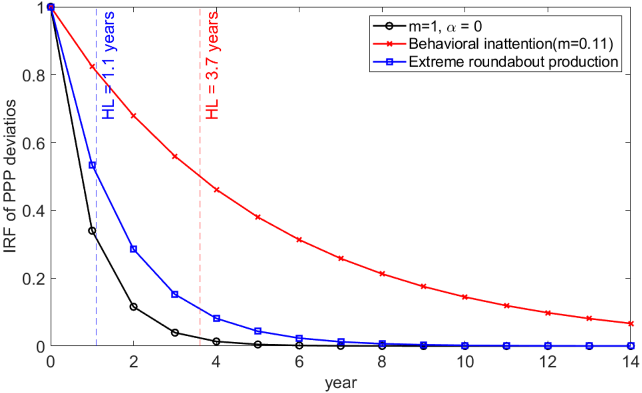
   - Then, marginal costs are insensitive to real wages

     $$\widehat{mc}_{Hit} = (1-r)\hat{w}_t$$

   - As r becomes closer to 1, marginal costs are insensitive to shocks even under full attention

Behavioral inattention VS. Roundabout production

- Comparing half-lives
- Behavioral inattention based on the estimated m VS. Extreme case of roundabout production ($\alpha = 0.99$)



- Much powerful in generating persistence

Thank you!

Households

- Domestic household solves

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t (\ln c_t - \chi n_t)$$

$$\begin{aligned} \text{s.t. } M_t + B_t &= W_t n_t + R_{t-1} B_{t-1} + (M_{t-1} - P_t c_{t-1}) + T_t + \Pi_t \\ M_t &\geq P_t c_t \end{aligned}$$

M_t: nominal money holding, B_t: nominal bond holding, W_t: nominal wage, R_t: nominal interest, P_t: price level, T_t: transfers, Π_t: profits, δ: discount factor

- Foreign household's problem is analogously defined except for the budget const.

$$\begin{aligned} \text{s.t. } M_t^* + \frac{B_t^*}{S_t} &= W_t^* n_t^* + \frac{R_{t-1}}{S_t} B_{t-1}^* + (M_{t-1}^* - P_{t-1}^* c_{t-1}^*) + T_t^* + \Pi_t^* \\ M_t^* &\geq P_t^* c_t^* \end{aligned}$$

S_t: nominal exchange rate

Households (2)

- FOC

$$\begin{aligned} \frac{W_t}{P_t} &= \chi c_t,\ \frac{W_t^*}{P_t^*} = \chi c_t^* \\ M_t &= P_t c_t,\ M_t^* = P_t^* c_t^* \end{aligned}$$

$$\begin{aligned} \frac{1}{R_t} &= \delta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-1} \frac{P_t}{P_{t+1}} \right] = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-1} \frac{S_t}{S_{t+1}} \frac{P_t^*}{P_{t+1}^*} \right] \\ q_t \frac{U_{c,t}}{U_{c,t}^*} &= q_{t-1} \frac{U_{c,t-1}}{U_{c,t-1}^*} = \cdots = q_0 \frac{U_{c,0}}{U_{c,0}^*} = 1 \end{aligned}$$

back

CES aggregators

- Home and Foreign (*)
- Consumption of good i

$$c_{it} = \left[\int c_{it}(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \; c^*_{it} = \left[\int c^*_{it}(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  - where z denotes the brand z of good i
  - $z \in [0, 1/2]$ is produced in home and $z \in (1/2, 1]$ is produced in foreign

- Aggregate consumption

$$c_t = \left[\int c_{it}^{(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \; c^*_t = \left[\int c_{it}^{*(\varepsilon-1)/\varepsilon} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

back

Resource constraint

- Production function

$$y_{it}(z) = a_{it} n_{it}(z)$$

  - $z \in [0, 1/2]$ are domestic firms

$$y^*_{it}(z) = a^*_{it} n^*_{it}(z)$$

  - $z \in (1/2, 1]$ are foreign firms

- Resource constraint

$$\begin{aligned} c_{it}(z) + (1+\tau) c^*_{it}(z) &= y_{it}(z) \text{ for } z \in [0, 1/2] \\ (1+\tau) c_{it}(z) + c^*_{it}(z) &= y^*_{it}(z) \text{ for } z \in (1/2, 1] \end{aligned}$$

  - Firms supply their goods to home and foreign cities

back

Price indexes for good i

- Under Calvo pricing,

$$\begin{aligned}\hat{p}_{it} &= \lambda(\hat{p}_{it-1} - \pi_t) + (1-\lambda)\hat{p}_{it}^{opt}(m_H, m_F) \\ \hat{p}_{it}^{*} &= \lambda(\hat{p}_{it-1}^{*} - \pi_t^{*}) + (1-\lambda)\hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*})\end{aligned}$$

- $\hat{p}_{it}^{opt}(m_H, m_F)$, $\hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*})$ are the weighted average of reset prices:

$$\begin{aligned}\hat{p}_{it}^{opt}(m_H, m_F) &= \omega\hat{p}_{Hi}(m_H) + (1-\omega)\hat{p}_{Fi}(m_F) \\ \hat{p}_{it}^{opt*}(m_F^{*}, m_H^{*}) &= \omega\hat{p}_{Fi}(m_F^{*}) + (1-\omega)\hat{p}_{Hi}(m_H^{*})\end{aligned}$$

where $1/2 < \omega < 1$ is the degree of home bias as a function of trade costs τ back

Proposition 2

Under the same assumptions in Proposition 1,

$$\rho_q \geq \lambda$$

provided $m \in (0, 1]$ *and* $\lambda \in (0, 1)$*.*

- Aggregate the LOP deviations

$$\ln q_{it} = \lambda \ln q_{it-1} + (1-m)(1-\lambda) \ln q_t + \lambda \varepsilon_t^n + \psi \varepsilon_{it}^r$$

  to get

$$\ln q_t = \rho_q \ln q_{t-1} + \rho_q \varepsilon_t^n,$$

  where

$$\rho_q = \frac{\lambda}{1-(1-m)(1-\lambda)}$$

- back

Proposition 3

Under the same assumptions in Proposition 1,

$$\rho_q \geq \rho_{qi}$$

provided $m \in (0,1]$, $\lambda \in (0,1)$, $\tau \in [0,\infty)$, $\varepsilon \in (1,\infty)$, *and* $\sigma_r/\sigma_n \in [0,\infty)$.

- The relationship btwn ρ_q and ρ_{qi} is

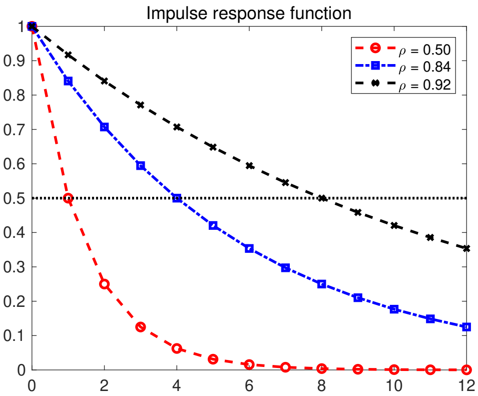
$$\rho_q = \left[\frac{1}{1-(1-m)(1-\lambda)\frac{A}{1+A}} \right] \rho_{qi}$$

where

$$A = \psi^2 \frac{1-\rho_q^2}{\rho_q^2(1-\lambda^2)} \left(\frac{\sigma_r}{\sigma_n} \right)^2$$

- back

Measuring persistence: Half-life



$$y_t = \rho y_{t-1} + e_t$$

- Half life is 1 year($\rho = 0.5$), 4 year($\rho = 0.84$), 8 year($\rho = 0.92$)

back

Rational inattention by Mackowiak and Wiederholt

- Consider the Rational Inattention (RI)
- Assume the same profit function
- Noisy information
  - Observe only signal $\hat{s} = \hat{w} + \epsilon$
  - where $\hat{w} \sim N(0, \sigma_w^2)$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$

$$\max_{\hat{p}_i} \mathbb{E}\pi(\hat{p}_i, \hat{w}) = \max_{\hat{p}_i} \mathbb{E}\left[-\frac{r}{2}(\hat{p}_i - \hat{w})^2\right]$$

  - The optimal price is the posterior belief of $\hat{w}$ given $\hat{s}$

$$\hat{p}_i = \mathbb{E}(\hat{w}|\hat{s}) = (1 - m^{RI})\mathbb{E}(\hat{w}) + m^{RI}\hat{s}$$

  - Namely

$$\hat{p}_i = \hat{p}_i(s) = m^{RI}\hat{w} + m^{RI}\epsilon, \text{ where } m^{RI} = \frac{\sigma_w^2}{\sigma_w^2 + \sigma_\epsilon^2}$$

Choice of information (1)

- In RI, firms choose the information strategy to maximize their profits with minus the cost of info. processing

$$\max_{f} \int_{\hat{w}} \int_{\hat{s}} \pi(\hat{p}_i(s), \hat{w}) f(\hat{w}, \hat{s}) d\hat{s} d\hat{w} - \mathcal{C}(f)$$

- Cost of info. processing $C(f)$ is represented by the mutual information:

$$\mathcal{C}(f) = \frac{c}{2}[H(\hat{w}) - H(\hat{w}|\hat{s})],$$

  where $H(\hat{w})$ is entropy of continuous random variable
- In general, the entropy $H(\hat{w})$ is defined by

$$H(\hat{w}) = -\int_{\hat{w}} g(\hat{w}) \log g(\hat{w}) d\hat{w}$$

  where $g(\hat{w})$ is pdf of $\hat{w}$

Choice of information (2)

- Under noisy info with normality, the choice of distribution is translated to the choice of uncertainty
- The max problem is to choose the posterior uncertainty $\sigma^2_{\hat{x}|\hat{s}}$

$$\max_{f} \mathbb{E}\left[-\frac{r}{2}(\hat{p}_i(s) - \hat{w})^2\right] - \frac{c}{2}[H(\hat{w}) - H(\hat{w}|\hat{s})]$$
$$= \max_{\sigma^2_{\hat{w}|\hat{s}}} \left[-\frac{r\sigma^2_{\hat{w}|\hat{s}}}{2} - \frac{c}{2}\log\left(\frac{\sigma^2_{\hat{w}}}{\sigma^2_{\hat{w}|\hat{s}}}\right)\right]$$

  where $\sigma^2_{\hat{w}|\hat{s}}$ can be reduced by lowering noise
- Trade-off
  - Benefit: acquire more accurate information to generate $\mathbb{E}[\hat{p}_i|\hat{s}]$
  - Cost: paying the cost of information processing

Choice of information (3)

- $\sigma^2_{\hat{w}}$ and $\sigma^2_{\hat{w}|\hat{s}}$ in the new max prob is related to m^{RI} in noisy info:

$$m^{RI} = \frac{\sigma^2_{\hat{w}}}{\sigma^2_{\hat{w}} + \sigma^2_{\hat{\epsilon}}} = 1 - \frac{\sigma^2_{\hat{w}|\hat{s}}}{\sigma^2_{\hat{w}}}$$

- Using $\sigma^2_{\hat{w}|\hat{s}} / \sigma^2_{\hat{w}} = 1 - m^{RI}$, the max problem becomes

$$\max_{m^{RI} \in [0,1]} \left[-\frac{r\sigma^2_{\hat{w}}}{2}(1 - m^{RI}) + \frac{c}{2}\log(1 - m^{RI}) \right]$$

- FOC implies

$$m^{RI} = \max\left[0, 1 - \frac{c}{r\sigma^2_w}\right]$$

▸ Behavioral inattention

The persistence of the PPP and LOP deviations

- For US-Canada with $m = 0.106$
- Monetary policy follows Taylor rule

	Model ($m = 1$)	Model ($m < 1$)	Data
Aggregate RERs (PPP deviations)			
Half-life	0.6	2.6	4.9
Good-level RERs (LOP deviations)			
Half-life	0.6	1.8	1.6

- Note: The unit of the HL is year
- Results are robust to specification of monetary policy

The persistence of the PPP and LOP deviations: Taylor rule

- For UK-Euro with $m = 0.134$
- Monetary policy follows Taylor rule

	Model $(m = 1)$	Model $(m < 1)$	Data
Aggregate RERs (PPP deviations)			
Half-life	0.6	2.4	2.4
Good-level RERs (LOP deviations)			
Half-life	0.6	1.6	1.2

- Note: The unit of the HL is year
- Results are robust to specification of monetary policy back