Noisy Foresight

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Teaching Slides

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Foresight

- Perfect foresight is a fundamental assumption in our theories
- ... but people fail at contingent reasoning (Charness and Levin, 2009; Esponda and Vespa, 2014)
- ... and nevertheless have to make choices

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How do people make decisions in the absence of perfect foresight?

Some Possibilities

- Game theorists have speculated about this:
  - future payoffs beyond some horizon are ignored (Jehiel, 1995) or considered random (Jehiel, 2001)
  - aggregate future payoffs in some boundedly rational way (Rampal, 2018; Ke 2019)

This Paper

- Design an *individual-decision* problem requiring foresight of decisions at future contingencies
  - rules out other-regarding preferences, reciprocity, strategic ambiguity, etc.
- Experiment: elicit behavior over a wide range of parameterizations
  - to go beyond showing failures
- Build and estimate potential models to see which performs best

Preview of Results

- Subjects do not forecast their *own* actions
  - rules out other reasons for backwards induction failures
- Best model of limited foresight:
  - mistaken beliefs (à la QRE)
    + discounting (cognitive attenuation)

Gneezy-Potters Task

Period 1 (Choose x)	Period 2		
$1-x$	Safe return	$R_1 = 1.5$	
x	Risky return	$R_F^+ = 3$	$p = .5$
		$R_F^O = 1$	$1-p = .5$

Our Task

Period 1 (Choose x)	Period 2 (Take outside option?)			
$1-x$	Safe return	$R_1 = 1.5$		
x	Risky return	$R_F^+ = 3$ $R_F^- = 0$	$p = .5$ $1-p = .5$	Outside option $R_F^O = 1$

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- Reduces to...

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x	Risky return	$R_F^+ = 3$ $\max\{R_F^-, R_F^O\} = 1$	$p = .5$ $1-p = .5$

Two-period Parameters

Decision Problem	Parameters				
	p	R_1	R_F^+	R_F^O	R_F^-
1a-1d	0.5	1.5	2.2	1	$\{0, .3, .7, 1\}$
2a-2d	0.2	1.5	4	1	$\{0, .3, .7, 1\}$
3a-3d	0.3	1.5	3	1	$\{0, .3, .7, 1\}$
4a-4d	0.2	1.5	3	1	$\{0, .3, .7, 1\}$
5a-5d (FOSD)	0.3	1	3	1	$\{0, .3, .7, 1\}$

- How much does (irrelevant) R_F^- matter?

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- How much does (irrelevant) R_F^- matter?
- Comparative statics: (R_F^+), (p), (R_1), and R_F^-
- $4 \times 5 = 20$ decision problems + 5 corresponding lottery tasks (measure risk aversion & comparison)
- attention check: FOSD lottery

Three-period Treatment

- Extend tasks to three periods
  - non-trivially reducible to two periods (and lottery) under rationality
- Some subtle comparison static results
- ... but broad conclusions are unchanged

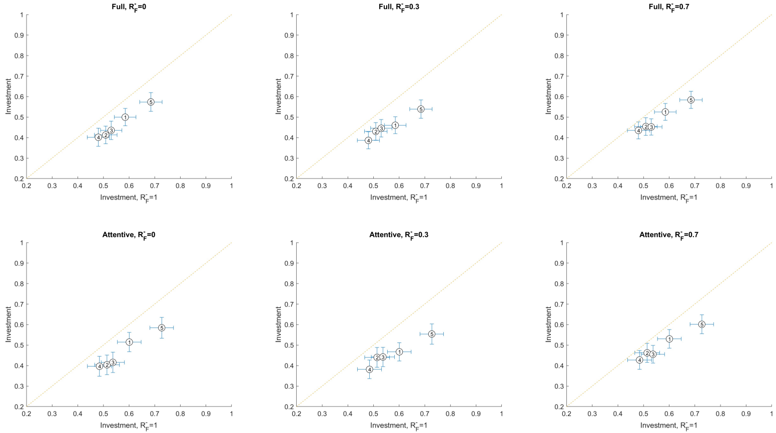
Implementation

- Lotteries follow decision problems (randomly ordered within each)
- Immediate feedback after each problem with 20 repetitions of similar tasks
- Run on Prolific, $n = 249$
- about 20min, $12/hour

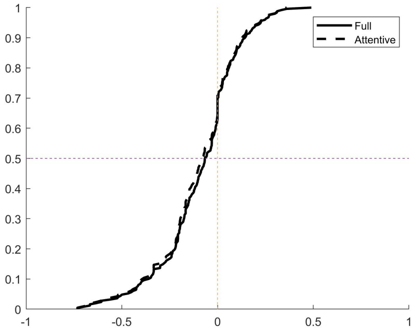
Sample Selection

- restrict to subjects that do not make dominated choices in final period (214/249)

Average Investments ($R_F^- = 1$ vs $R_F^- = 0$)



Heterogeneity



- Individual differences, averaged across decision problems

Other Comparative Statics

- Other comparative static predictions (p, R_1, R_F^+).
  - 76% of investment-decision pairs respect prediction (84% in the Satisfy FOSD subsample)
- $\Rightarrow$ vast majority of subjects respond to the parameters other than R_F^- as standard theory would predict
  - subjects apply foresight, albeit imperfectly

Learning

- Non-parametric: figures look almost identical in second half of data
- Parametric : investment does not increase over time except for handful of subjects

Main Takeaways

1. Irrelevant return (R_F^-) matters
2. But other comparative statics (p, R_1, R_F^+) are as one would expect (necessary to build any theory of bounded rationality)
3. Little learning about own hypothetical actions, even after making rational choices in final period
4. Results are at least as strong among those paying attention (satisfy FOSD)

Models (1)

- Results already rule out some models put forth in the literature
- Future payoffs are:
  - not ignored
  - not random
  - not aggregated without considering probability (i.e., Lapacian)

Models (2)

- Rational
- Naïve (ignore outside option)
- Cursed (Eyster and Rabin, 2005)
  - allow for 'partially cursed'
- Tremble (ambiguity)
  - believe mistakes occur with constant probability ϵ
- Noisy self
  - believe mistakes proportional to utility difference (à la QRE)
- Generalized mean (Ke, 2019)

$$u^+ = \left(\frac{1}{2}\left(u(xR_F^+ + (1-x)R_1)\right)^{\gamma} + \frac{1}{2}\left(u(xR_F^O + (1-x)R_1)\right)^{\gamma}\right)^{1/\gamma}$$

Cognitive Attentuation

- Payoffs in the future may be attenuated due to cognitive uncertainty
- For each model, consider variation with exponential discounting parameter ($\delta \leq 1$)

Estimation

- Assume CRRA utility, $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$
- First estimate risk-aversion from lottery tasks using OLS
- Then estimate model using investment choices
  - assume logit noise
  - continuous investment choice is treated as discrete on grid of 0.01

Comparing Models

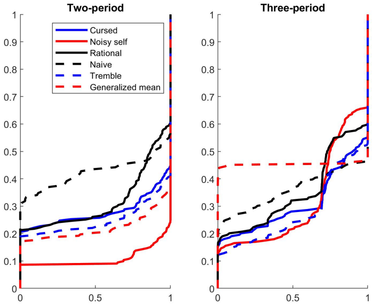
- Nested models:
    - likelihood ratio test
- Non-nested models:
    - Vuong test

Model Comparison Results

Model	Rational	Naïve	Cursed	Noisy Self	Tremble	Generalized mean
Rational	-	0.07	-	-	-	-
Naïve	**0.16**	-	**0.53**	0.23	**0.53**	**0.50**
Cursed	**0.22**	0.08	-	0.02	0.01	0.07
Noisy self	**0.55**	**0.22**	**0.46**	-	**0.38**	**0.37**
Tremble	**0.24**	0.08	**0.15**	0.02	-	0.08
Generalized mean	**0.35**	0.12	**0.16**	0.05	**0.08**	-

- Noisy self dominates in three-period as well

Cognitive Attenuation (δ or δ^2)



- improves fit significantly in 10/12 cases

Potential concern

- NS model fits best because it interacts with the logit noise structure we impose in estimation?
- Or, overfits the data more than other models?
- Solution: predict the first-period decisions for each decision problem at the individual level using estimates obtained from estimation on the other (20-1=19) decision problems

Measuring Out-of-sample Fit

- Calculate completeness measure (Fudenberg et al 2022a, Fudenberg et al 2022b) for each model, f_θ, for individual i as:

$$CV_i(f_\theta) = \frac{\mathcal{E}(f_{baseline}) - \mathcal{E}(f_\theta)}{\mathcal{E}(f_{baseline}) - \mathcal{E}(f_{perfect})}$$

- $\mathcal{E}()$ is the out-of-sample mean-squared prediction error across all decision problems (no logit assumption)
- Baseline model, f_{naive}, is taken to be the rational, risk-neutral prediction
- Perfect model, $f_{perfect}$, is taken to be the actual choice (so that $\mathcal{E}(f_{perfect}) = 0$)

Completeness Results

Table: Model Completeness

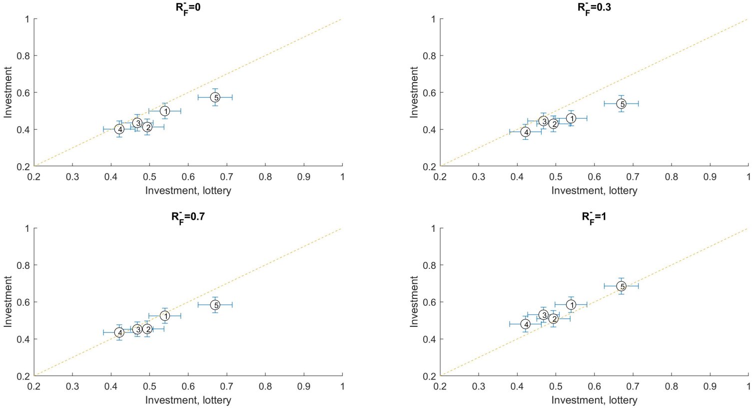
Model	Two-period No Cog. Att.	Two-period Cog. Att.
Rational	0.54	0.52
Naïve	0.36	0.29
Cursed	0.54	0.54
Noisy self	0.60	0.62
Tremble	0.54	0.53
Generalized mean	0.50	0.50

- Noisy self predicts best out-of-sample in three-period as well

Conclusion

- Contingent reasoning failures extend to one's 'future self'
    - experience does not help
- Both noisy-self (proportional mistakes) for forming beliefs and cognitive attentuation for payoffs matter
    - can noisy perception integrate these two features?
- More work needed to model bounded rationality in sequential problems

Investments vs Lotteries (Two-period)



Risk Estimates