Labor Market Dynamics and Growth

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- Explore interaction between endogenous growth, business cycles and labor market frictions.

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Gives:

- [Standard] Gross labor market flows, endogenous wages, tightness, unemployment etc
- [New] Endogenous growth via imitation

Implications

Standard search model (DMP):

- Labor market tightness equilibrates job creation condition
  - Tightness raises wages and lowers vacancy filling rate.
- Implication: *average* match productivity determine tightness

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This model:

- Labor market tightness equilibrates wage in *marginal* job
- Creative destruction equilibrates market for job creation
- *Marginal* match value responds strongly to productivity
  $\Rightarrow$ New resolution of the Shimer puzzle

A Stationary Model

The Environment—Diamond-Mortensen-Pissarides

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- Exogenous separations occur at Poisson rate δ.
- Exogenous growth of productivity at rate μ

The Environment—Endogenous Growth

Match quality

- After meeting, workers and firms draw their productivity Z from $\Phi(Z, t)$.
- Where $\Phi(Z, t)$ is the existing distribution of matches in the economy.
  - Technology embodied in jobs and diffusion of knowledge in job creation

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- **Note.** Corresponds to the DMP model if the distribution $\Phi(Z, t)$ is constant over time.

The Environment—income

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- Vacancy cost

$$K(t) = k_0 \overline{Z}(t)$$

- Flow income in unemployment

$$B(t) = b_0 \overline{Z}(t)$$

Model - Value Functions

Worker

$$rV_e(Z,t) = W(Z,t) + \delta(V_u(t) - V_e(Z,t)) + \mu Z \frac{\partial V_e(Z,t)}{\partial Z} + \frac{\partial V_e(Z,t)}{\partial t}$$

$$rV_u(t) = B(t) + \theta(t)q(\theta(t)) \int (V_e(Z,t) - V_u(t)) d\Phi(Z,t) + \frac{\partial V_u(t)}{\partial t}$$

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Firm

$$rJ(Z,t) = pZ - W(Z,t) - \delta\left(J(Z,t) - V_v(t)\right) + \mu Z \frac{\partial J(Z,t)}{\partial Z} + \frac{\partial J(Z,t)}{\partial t}$$

$$rV_v(t) = -K(t) + q(\theta(t)) \int (J(Z,t) - V_v(t)) d\Phi(Z,t) + \frac{\partial V_v(t)}{\partial t}$$

Normalization

- Model admits a normalization, whereby all variables are scaled by the lowest productive firm.
- For wage $W(Z,t)$ for example,

$$W(Z,t) = M(t)w(z,t) \qquad \text{where,} \quad z \equiv Z/M(t)$$

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- Surplus solves

$$S(z,t) \equiv J(Z,t) - V_v(t) + V_e(Z,t) - V_u(t)$$

  Normalized surplus of a match, $s(z,t) \equiv S(z,t)/M(t)$, is given by,

$$(r+\delta-g(t))s(z,t) = pz - b - \theta(t)q(\theta(t))\int_1^{\infty}(v_e(z,t)-v_u(t))d\Phi(Z,t)$$
$$-\frac{\partial s(z,t)}{\partial z}z(g(t)-\mu) + \frac{\partial s(z,t)}{\partial t}$$

Equilibrium - definition

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- And (i) the free entry condition holds; (ii) matches separate efficiently; (iii) wages are the solution to Nash bargaining.

Equilibrium conditions

Free entry

- Firms continue to post vacancies until $v_v = 0$

$$\frac{k}{q(\theta)} = (1-\beta)\int_{M(t)}^{\infty} s(Z/M(t))d\Phi(Z,t)$$

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Endogenous growth

- Optimal exit by firms

$$(g-\mu)s(1) = (g-\mu)s'(1) = 0$$

where, $g \geq \mu$, $\theta \geq 0$ and $s(z) \geq 0$

Kolmogorov Forward Equation

$$\frac{\partial \Phi(Z,t)}{\partial t} = \underbrace{\Phi(Z,t)E(t)}_{\text{entrants}} - \underbrace{\Delta(t)}_{\text{endogenous separations}} - \underbrace{\delta \Phi(Z,t)}_{\text{exogenous separations}} - \underbrace{\mu Z \frac{\partial \Phi(Z,t)}{\partial Z}}_{\text{productivity drift}}$$

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- It can be shown that,

$$\Psi(z) = 1 - \left(\frac{1}{z}\right)^{\alpha} \qquad \text{where,} \quad z := \frac{Z}{M(t)}$$

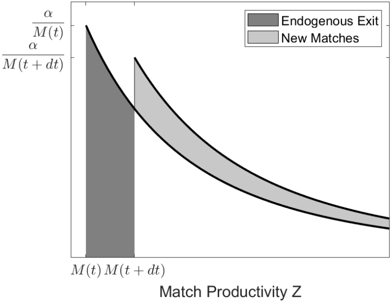
and

$$\Delta(t) = \alpha(g - \mu) \qquad \text{where,} \quad g := \frac{M'(t)}{M(t)}$$

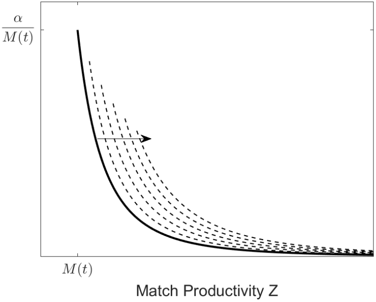
Evolution of Match Productivity

- Discrete time representation...

Entering and Exiting Matches



Gradual Evolution of $\Phi(Z, t)$



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No endogenous growth, $g = \mu$

Endogenous growth, $g > \mu$

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- The growth rate g is pinned down by free entry such that,

$$g = -\frac{r+\delta-\alpha\mu}{\alpha-1} + p\frac{q(\bar{\theta})}{k}\frac{1-\beta}{(\alpha-1)^2}$$

Amplification: a first glance

- Log-linearizing equilibrium condition wrt p where $b = \frac{B(t)}{M(t)}$ gives
- For $g = \mu$ (DMP)

$$\frac{d\log(\theta)}{d\log(p)} = \frac{\frac{\alpha}{\alpha-1}p}{\frac{\alpha}{\alpha-1}p - b} \underbrace{\frac{\beta\theta q(\theta) + r + \delta - \mu}{\beta\theta q(\theta) + \eta(r + \delta - \mu)}}_{\approx 1}$$

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- Marginal jobs determine tightness $\rightarrow$ much more amplification
  - Older view of demand and relates to Nagypal and Mortensen (2007), Hagedorn and Manovskii (2008) and Elsby and Michaels (2013)

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- Calibration suggest that $\frac{\partial g}{\partial p} < 0$ so labor productivity is *moderated* by growth.
- Require larger shocks to p to generate the same movements in cyclical labor productivity.

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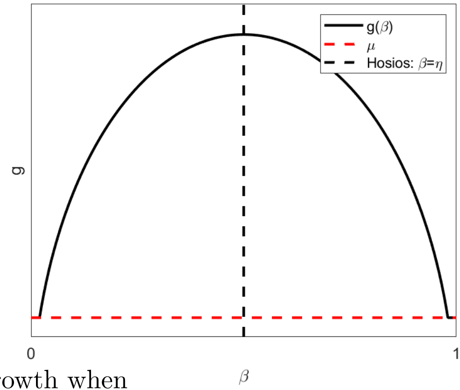
- One small change $\rightarrow$ very different economics

Growth rate & worker bargaining



- Endogenous growth when

$$\frac{pAk^{\eta-1}}{(p-b)^{\eta}(\alpha-1)}\,(1-\beta)^{1-\eta}\,\beta^{\eta} > r+\delta-\mu$$

  - For intermediate values of β

- Growth is maximized when $\beta = \eta$ (akin to Hosios)

The Model with Aggregate Shocks

Aggregate Shocks

- An aggregate state variable $a \in [0, 1]$
- Such that labor productivity is given by $p(a)$, where $p'(a) > 0$
- Following a Poisson shock χ draw new aggregate state a' from $\gamma(a, a')$ (locally uniform)

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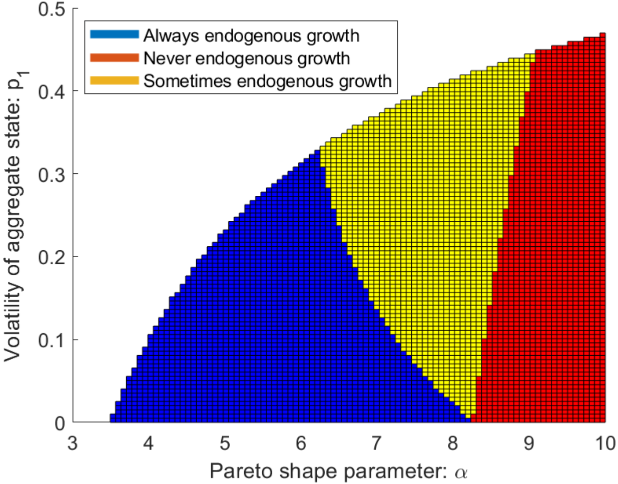
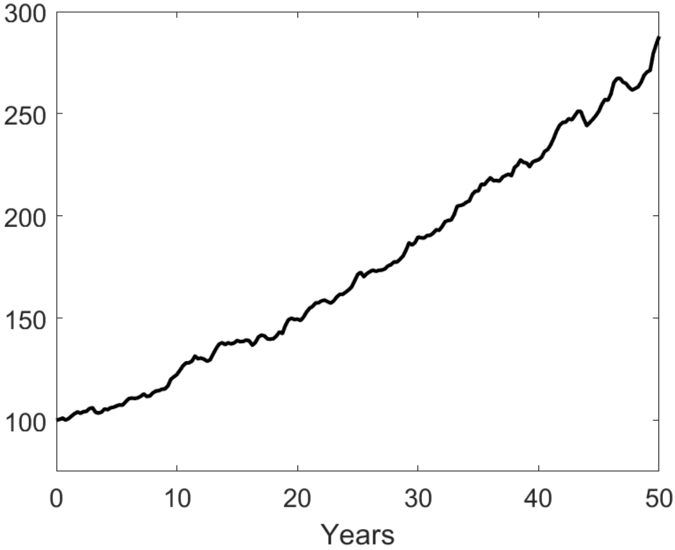
Calibration see here.

Class of Equilibria

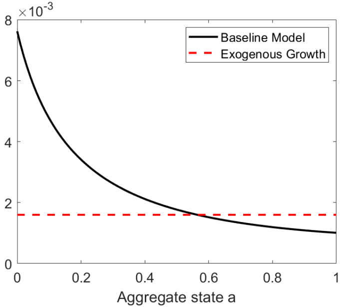


Evolution of output - baseline model

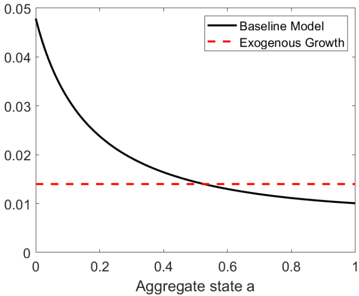


Creative destruction

Growth rate $g(a)$



Separation rate



- Creative destruction higher in recessions.

Amplification

	Untargeted			Targeted
	u	f	s	lp
Data				
Standard deviation	0.10	0.074	0.064	0.0089
Quarterly autocorrelation	0.94	0.83	0.75	0.74
Baseline model				
Standard deviation	0.080 (0.011)	0.056 (0.0066)	0.049 (0.0059)	0.0089 (0.0010)
Quarterly autocorrelation	0.86 (0.028)	0.71 (0.049)	0.71 (0.050)	0.72 (0.049)
Exogenous growth (DMP)				
Standard deviation	0.0080 (0.0015)	0.036 (0.0043)	0 (−−−)	0.0088 (0.0010)
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All series are quarterly, seasonally adjusted, and reported as log deviations from an HP trend with smoothing parameter 1600. The standard deviation from 1000 model resimulations are reported in parenthesis.

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- Data: Much more variation in unemployment and job finding rate compared to measured labor productivity

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- DMP: Similar variation in unemployment and job finding rate to measured labor productivity (Shimer, 2005)

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- Baseline model: Captures most of the variation in unemployment and job finding rate given measured labor productivity (Shimer, 2005)

Concluding Remarks

- One small perturbation to an otherwise standard model.
  - Entrant's productivity is drawn from the distribution of incumbents.
- Resulting model generates
  - Endogenous growth.
  - Different economics.
  - Greater amplification of productivity to unemployment (consistent with data).

—— Thanks for Listening ——–

Parameterization

- Assume the log of p is linear in a

$$\log(p(a)) = p_0 + p_1(a - 0.5) \quad \text{where, } a \in [0,1]$$

- Following a χ shock new state a' drawn from

$$\gamma(a, a') = \begin{cases} \frac{1}{\epsilon} & \text{if } a' \in [a(1-\epsilon), a(1-\epsilon) + \epsilon] \\ 0 & \text{otherwise} \end{cases}$$

  - Bounded and stationary.
  - Exhibits mean reversion.
  - Analytical formulas for moments

$$\text{Var}(\log(p(a_t))) = p_1^2 \frac{\epsilon}{12(2-\epsilon)},$$
$$\mathbb{E}(\log(p(a_t))|a_0) = p_0 + p_1 \exp\left[-\chi\epsilon t\right](a_0 - 0.5),$$

return

Calibration

- Calibrate two models
  - Baseline model: as described
  - Exogenous growth: $\alpha \to \infty$
- Under exogenous growth
  - Degenerate match quality distribution.
  - All growth through deterministic drift μ
- Models calibrated to US labor market 1990-2019, inclusive.

Targeted Moments

Table 1: Aggregate Moments

Parameters Baseline	Exog. Growth	**Moments**	**Source**	**Value** Data	Baseline	Exog. Growth
	$\alpha \to \infty$	By assumption				
$\alpha = 5.71$	$\mu = 0.0016$	Growth rate	BLS	0.0016	0.0016	0.0016
$A = 0.41$	$A = 0.41$	Job finding rate	CPS	0.253	0.253	0.253
$\mu = 0.0005$	$\delta = 0.014$	Separation rate	CPS	0.014	0.014	0.014
$k_0 = 4.54$	$k_0 = 4.55$	Mean tightness	CPS+JOLTS	0.581	0.581	0.581
$b_0 = 0.71$	$b_0 = 0.71$	Unemployment benefit/mean output	Hall and Milgrom (2008)	0.71	0.71	0.71
$\beta = 0.042$	$\beta = 0.042$	Pass-through $\left(\frac{\partial \log(w)}{\partial \log(z)}\right)$	Card et al. (2018)	0.05	0.05	0.05
$\delta = 0.007$	—	Separation rate (high wage)$^{+}$	Mueller (2017)	0.007	0.007	—

$^{+}$ The separation rate of the high wage is defined as the separation rate of those earning above the median wage.

Targeted Moments

Table 2: Dynamic Moments

Parameters Baseline	Exog. Growth	**Moments**	**Source**	**Value** Data	Baseline	Exog. Growth
$\epsilon = 0.16$	$\epsilon = 0.13$	st. dev. of labor productivity*	BLS	0.0089	0.0089	0.0089
$\chi = 0.40$	$\chi = 0.63$	autocorrelation of labor productivity*	BLS	0.75	0.71	0.71
$p_0 = -9.8 \times 10^{-5}$	$p_0 = -8.9 \times 10^{-5}$	normalize mean productivity	—	1	1	1
$p_1 = 0.16$	$p_1 = 0.19$	st. dev. of first difference log labor productivity	BLS	0.0067	0.0067	0.0067
$\eta = 0.13$	—	relative st. dev. of finding rate / separation rate*	CPS	1.16	1.16	—

All of the moments in this table are the mean computed after resimulating the model 1000 times. For moments indexed by a *, the underlying series has been successively logged and detrended using an HP filter with a smoothing parameter of 1600.

return