Reconciling Relational Contracting and Hold-up: A Model of Repeated Negotiations

Susanne Goldlücke (Konstanz) and Sebastian Kranz (Ulm)

JEEA Teaching Material

Motivating Example: Repeated Principal Agent Game

- Each period: Agent A chooses effort $e \in [0, \bar{e}]$ or boycots principal

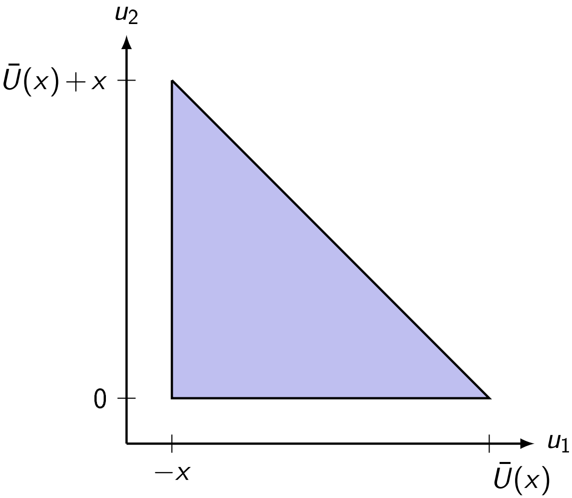
stage game payoffs	principal	agent
agent works	e	-$k(e)$
boycot	$-x$	0

- $k(e)$ is smoothly increasing and convex with $k(0) = 0$, $\bar{e}$ is efficient
- We call $x \geq 0$ principal's vulnerability
- At the beginning of each period players can voluntarily transfer money
  - risk neutral, unlimited liquidity
- Infinitely often repeated, discount factor $\delta \in [0, 1)$

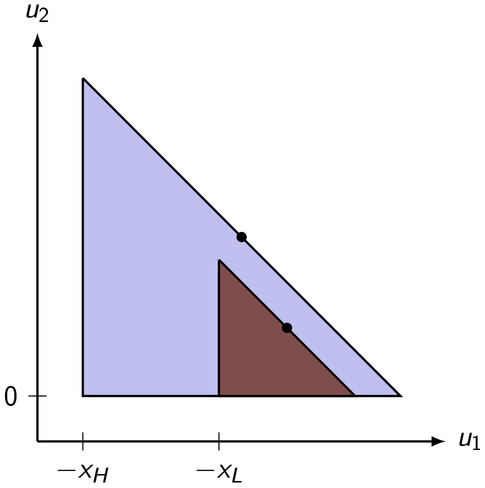
Relational Contracts

- Relational contract: A Pareto-optimal SPE of infinitely repeated game with transfers.
- Here, e.g.
  - Agent chooses maximal implementable effort e^* on equilibrium path
  - Principal pays agent a transfer next period only if e^* was chosen.
  - If principal deviates, agent boycots forever as punishment.
- e^* increases in δ and principal's vulnerability x

Set of (average discounted) SPE Payoffs



SPE Payoffs for low and high vulnerability



blue: high vulnerability, red: low vulnerability

Endogenous Vulnerability

- Assume game begins in an initial state x_0 in which the principal chooses her vulnerability:
  - Forever low vulnerability x_L or forever high vulnerability x_H
  - no costs
- Stage game now depends on an endogenous state x: we have a discounted dynamic game (also called *stochastic game*).
- Question: Under which conditions will the principal make herself highly vulnerable?

Vulnerability Paradoxon of Pareto-Optimal SPE

If the first-best effort $\bar{e}$ cannot be implemented with low vulnerability x_L then in **every** Pareto-Optimal SPE the principal makes herself highly vulnerable.

- Intuition: Higher vulnerability $\rightarrow$ harsher punishment possible $\rightarrow$ better incentives on equilibrium path.
- But what about hold-up? Should principal not worry about exploitation?
  - Hold-up is ruled out simply by assumption in Pareto-Optimal SPE.
- We belief more plausible is a trade-off between efficiency gain and risk of hold-up.
- Vulnerability Paradoxon is a key motiviation for our paper.

Main contributions

- Introduce "Repeated Negotiation Equilibrium" (RNE)
  - Relational contracts are newly negotiated from time to time
  - Refinement of SPE
  - Puts "hold-up" into relational contracts
- Many examples of dynamic games where Pareto-Optimal SPE are unintuitive but RNE model natural trade-offs.
  - Repeated games: RNE essentially just boil down to lower discount factor.

Related Literature

- Renegotiation-Proofness, e.g. Farell & Maskin (1989), Levin (2002), Goldlücke & Kranz (2013):
  - Key idea: Punishment should not be Pareto-dominated.
  - Has little bite in games with monetary transfers:
    - Allow punished player to settle punishment by paying a fine and continue afterwards as on equilibrium path
    - Every Pareto-optimal SPE payoff can be implemented with a renegotiation proof (strongly optimal) SPE.
- Miller & Watson (2013): Contract Equilibria
  - Does not solve Vulnerability Paradoxon either

A Simple Investment & Trade Game

- Period 1:
  - Firms $i = 1, 2$ choose investment in common infrastructure $e_i \in [0, 1]$
- Period 2:
  - Firms can trade with each other. Surplus from trade

$$S(e) = \frac{3}{2} \cdot (e_1 + e_2)$$

  - Assume $\delta > \frac{2}{3}$. Hence $e_1 = e_2 = 1$ is first best optimal
  - Surplus $S(e)$ is split via a Nash demand game: firms simultaneously announce demanded share $d_i \in [0, 1]$

$$\pi_i(e, d) = \begin{cases} d_i S(e) & \text{if } d_1 + d_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Note: This is a special case of a discounted dynamic game. Just fix payoff to 0 after period 2.

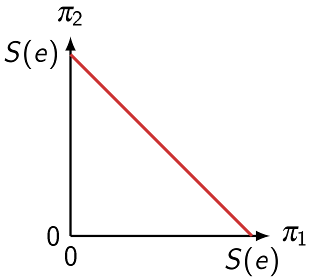
Simple Investment & Trade Game

Stage game equilibria in period 2

- Every possible split of the full surplus $S(e)$, e.g.

$$d = (0.1, 0.9)$$

  is a stage game Nash equilibrium in period 2.
- Same result with many other formulations of bargaining game in period 2. Famous (but not robust) exception: Rubinstein bargaining.



Continuation payoffs in period 2

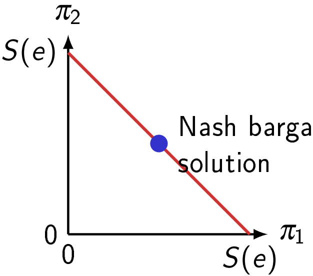
Simple Investment and Trade game

Hold-up Models:

- Behavioral assumption: In period 2, $S(e)$ is always split according to Nash bargaining solution

$$\pi_i = \frac{1}{2} S(e)$$

- A hold-up problem arises:
  - One extra unit of investment changes own payoff by $\delta \cdot \frac{1}{2} \cdot \frac{3}{2} - 1 < 0$
  - No player invests (essentially a public goods dilemma)

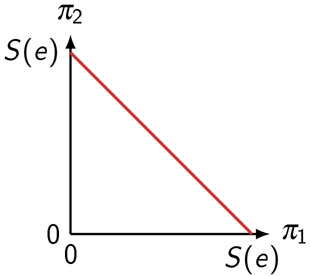


Continuation payoffs in period 2

The Investment and Trade game

Relational Contracting:
Pareto-optimal SPE

- Split of surplus will depend on investments:
  - Split surplus equally if both players invest 1.
  - If a player unilaterally invests less than 1, the other player gets whole surplus.
- Can always implement first-best investments.
- No hold-up problem!

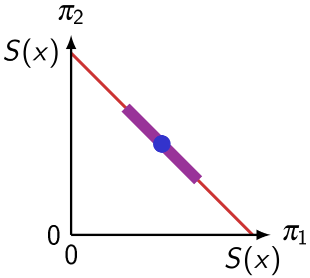


Continuation payoffs in period 2

The Investment and Trade game

Hold-Up or Relational Contracting: What shall we assume?

- Ellingsen and Johannesson (2004) study similar games in experiments
  - results suggest intermediate cases
- Our concept allows to model a continuum of intermediate cases

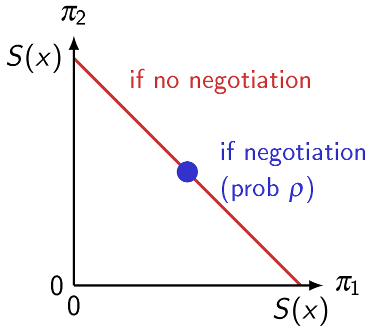


Continuation payoffs in period 2

The Investment and Trade game

Our concept

- Exogenous probability $\rho \in [0, 1]$ that relational contract is newly negotiated at beginning of each period
- Nash Bargaining:
  - expected payoffs $= \frac{1}{2} S(e)$
- If no new negotiation:
  - old relational contract stays in place and continuation play depends flexibly on history



The Investment and Trade game

Our concept: Repeated Negotiation Equilibria (RNE)

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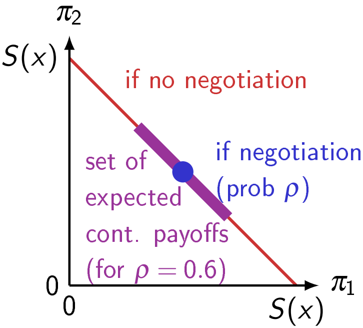
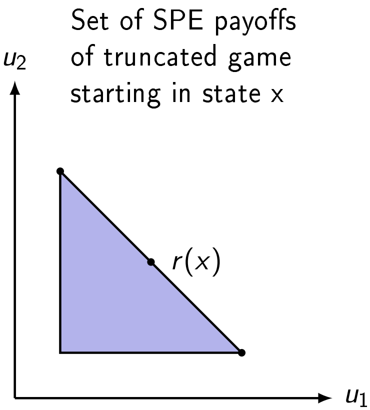


RNE for General Discounted Dynamic Games

- Repeated Negotiation Equilibria (RNE) is a refinement of SPE in discounted dynamic games with transfers and public correlation device
- Exogenous probability $\rho \in [0,1]$ that continuation equilibria are newly negotiated at beginning of a period.
- Negotiation payoffs $r(x)$ denote expected continuation payoffs if there is new negotiation in state x
  - Negotiation outcome only depends on state x, not on any other aspect of history

Nash Bargaining Solution and RNE

- Take future negotiaton payoffs r as given. Specify for each state x a "truncated" game $\Gamma(x, r)$:
  - stops with probability ρ each period
  - if it stops in state x, it grants fixed payoffs of $r(x)$ forever
- RNE: $r(x)$ must split Pareto-optimal SPE payoff of truncated game given r according to Nash Bargaining solution, with worst equilibrium payoffs as disagreement point.



Limit Cases

- Limit cases:
  - $\rho = 1$: RNE is a Markov Perfect Equilibrium (if there is a unique MPE)
  - $\rho = 0$: RNE is a Pareto-optimal SPE

Repeated Games

- In a repeated game negotiation is like a termination and restart of relationship.
- Relevant is adjusted discount factor: $\tilde{\delta} = (1-\rho)\delta$
- RNE always exist and have unique negotiation payoff:

$$r_i = \tilde{v}_i(\tilde{\delta}) + \beta_i(\tilde{U}(\tilde{\delta}) - \sum_{j=1}^{n} \tilde{v}_j(\tilde{\delta}))$$

  where $\tilde{U}(\tilde{\delta})$ and $\tilde{v}_j(\tilde{\delta})$ are maximal joint payoff and minimal punishment payoff of repeated game with discount factor $\tilde{\delta}$.
- Reduced discount factor already accounts for hold up in repeated games.

Blackmailing Game: No RNE exists

- 2 players, 2 states, $\rho > 0$
- Initial state x_0:
  - Payoffs $(0, 1)$.
  - Player 1 (blackmailer) can reveal harmful information about player 2 $\rightarrow$ moving permanently to state x_1
- State x_1:
  - Payoffs $(0, 0)$
  - no more actions $\rightarrow$ $r(x_1) = (0, 0)$

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- In the truncated game with $r_1(x_0) = 0$ blackmailer can extort money by credible threat to reveal information.
  - Nash bargaining solution would give blackmailer a negotiation payoff of $r_1(x_0) = \frac{1}{2}\delta$

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  - Nash bargaining solution would give blackmailer a negotiation payoff of $r_1(x_0) = \frac{1}{2}\delta$
- But if $r_1(x_0) > 0$, it is not anymore incentive compatible to reveal information. No money can then be extorted!

Weak RNE

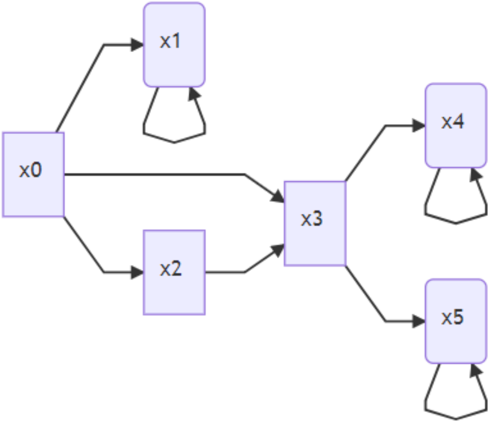
- Key idea:
  - In negotiations, player i randomly becomes dictator with probability β_i.
  - Dictator must pick her best SPE payoff of truncated game $\Gamma(x, r)$ but can ignore SPE payoffs that are not stable.
- Stability definition:
  - A SPE payoff u of a truncated game $\Gamma(x, r)$ is **stable** if there exists an ε-ball D^ε around r and a continuous function $f : D^\varepsilon \to \mathbb{R}^n$ such that $f(r) = u$ and $f(\tilde{r})$ is a SPE payoff of $\Gamma(x, \tilde{r})$ for all $\tilde{r} \in D^\varepsilon$.
  - In a repeated game every SPE is stable.

Weak RNE

- General existence result for weak RNE in stochastic games with transfers and public correlation device.
- In repeated game: Unique weak RNE payoff equal to RNE payoff
- Note: General existence result for weak RNE has probably little relevance for applied work.
  - Characterization of mixed strategy weak RNE payoff set for general stochastic games can become quite complicated.

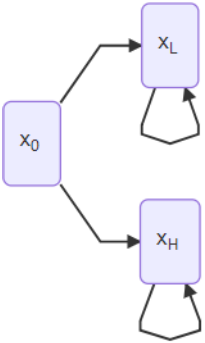
Strongly Directional Games

- Strongly directional game: finite number of states and only terminal states can be repeatedly visited. Example:

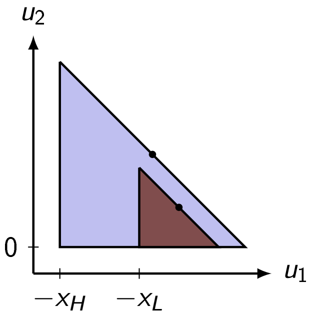


- RNE always exist. Unique RNE payoff. Fast numerical algorithm.

Back to the Vulnerability Paradoxon



RNE for fixed $\rho > 0$ and limit $\delta \to 1$: Principal chooses vulnerability $x \in \{x_L, x_H\}$ that would grant her a higher negotiation payoff in the repeated game with discount factor $\tilde{\delta} = (1-\rho)\delta$ and fixed vulnerability x .

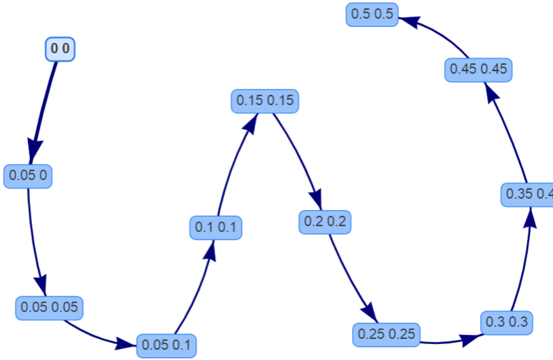


Variation of Vulnerability Paradox

- Variation of previous principal-agent example:
  - Both principal and agent can boycot.
  - Vulnerability of each player i can take 11 levels $x_i \in \{0, 0.05, 0.1, ..., 0.5\}$.
  - Each player starts with $x_i = 0$, but can increase vulnerability in each period to any higher level. Not possible to decrease.
- Assume that vulnerability can be changed only in the first $T = 1000$ periods.
  - We then have a strongly directional game where for $t \leq T$, a state is described by (t, x_1, x_2).
- Solve numerically, with cost $k(e) = -\frac{1}{2}e^2$ and effort from grid $e \in \{0, 0.01, 0.02, ..., 1\}$, adjusted discount factor $\tilde{\delta} = 0.25$.

Optimal SPE:

Repeated Negotiation ($\rho = 0.7$)



Both players make themselves immediately fully vulnerable.

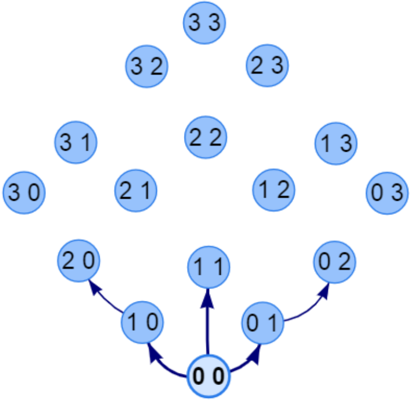
Gradual increase of mutual vulnerability.

Arms Race Example

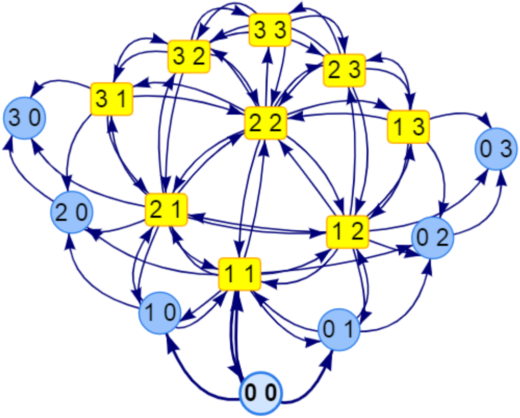
- Two countries that can perform costly investment into weapons
- State $x = (x_1, x_2)$ where $x_i = \{0, 1, ..., \bar{x}\}$ denotes weapons arsenal of country i.
    - Country i can increase or decrease x_i by one unit. Investment costly, not always successful
    - Maintanance costs $c_m \cdot x_i$ every period
    - Country i can attack other country and inflict harm proportional to x_i. Attack is costly.
- No direct gain from using weapons, but possibly can extract transfers by threat to use them.
- Only one randomly chosen country can act in a period.

Arms Race Example

- In no Pareto-optimal SPE weapons are bought
  - Reason: It is an SPE that players never make transfers and simply ignore any threat to use weapons.
- In no MPE weapons are bought or used
  - Reason: Not credible to pay cost of attack, since it cannot induce future payments.
- In RNE weapons can be bought. Numerical example on next slide.



Transitions in an RNE



Variation of game: Attacks can destroy other player's weapons

Summary

Key point:

- Repeated Negotiation Equilibria (RNE) account for hold-up concerns and role of bargaining positions in relationships with long term decisions
  - Pareto-optimal equilibria and existing (re-)negotiation refinements often do not

Future research:

- Alternative disagreement point than worst continuation equilibrium.

Appendix: Critical Negotiation Probability

- Relational contracting literature often computes a **critical minimal discount factor** $\bar{\delta}$ that is required to implement first-best actions
- We propose **critical maximal negotiation probability** $\bar{\rho}$ as an alternative
- Investment and Trade game:
  - First best investments $e_1 = e_2 = 1$ can be implemented if and only if

$$\rho \leq 2(1 - \frac{2}{3}\delta^{-1}) \equiv \bar{\rho}$$

Problem of Critical Discount Factors

- Critical minimum discount factors $\bar{\delta}$ make less sense in discounted dynamic games
  - First best can change in δ
- Investment and Trade game:
  - In a SPE full effort $e_1 = e_2 = 1$ can only be implemented if $\delta \geq \frac{2}{3}$.
  - But if $\delta \leq \frac{2}{3}$, also the first-best effort changes to $e_1 = e_2 = 0$.
  - Hence for every discount factor δ the first best can be implemented in a SPE