Risk Perception: Measurement and Aggregation

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JEEA Teaching Materials

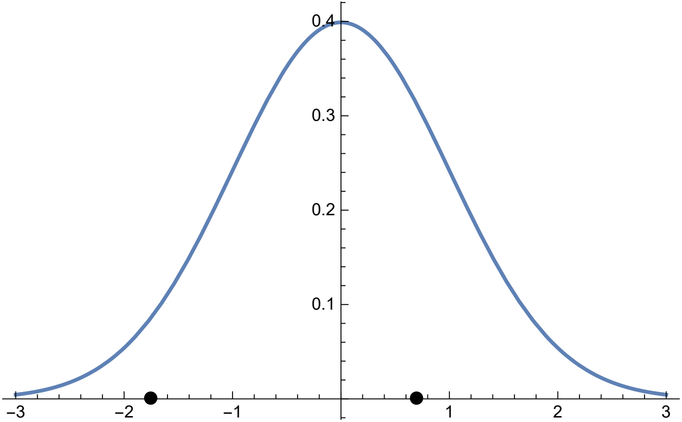
Nonlinear Measurement Scales

physical instruments often use nonlinear measurement scales

- this improves precision at some range of inputs
- at the expense of precision at other values

psychophysics literature extends this to human perception

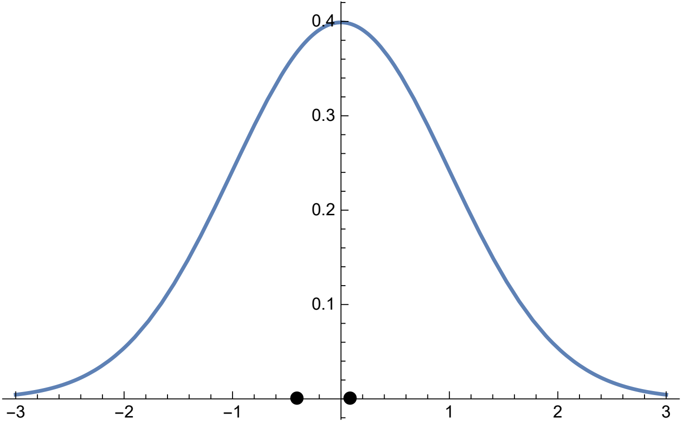
- Kahneman & Tversky '79 use this to justify S-shaped utility pick one of the two draws:



Formalization

Robson '01, Netzer '09

pick one of the two draws:



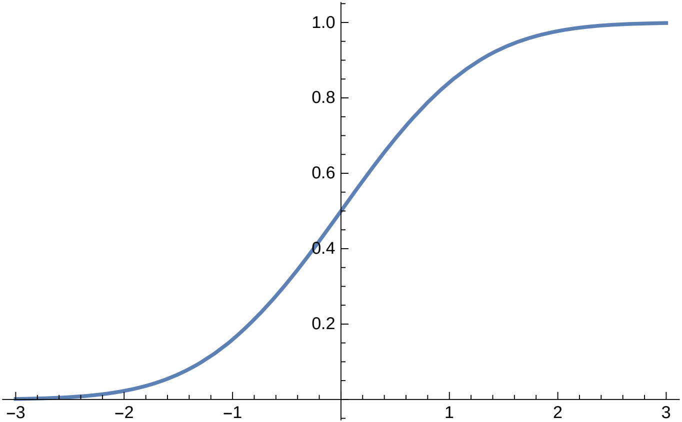
Formalization

Robson '01, Netzer '09

encode reward r_i as $m(r_i) + \varepsilon_i$

choose your encoding function m

optimal encoding function as noise vanishes



Our Contribution

Robson '01, Netzer '09:

- perception of one-dimensional inputs
- encoding function $\sim$ hedonic as opposed to Bernoulli utility
- vanishing implications for choice

this paper:

1. exogenous perception $\Rightarrow$ behavior
   - coarse model $\Rightarrow$ perception-driven risk attitudes
   - well-specified model $\Rightarrow$ risk-neutrality
2. optimal perception of lotteries
   - microfounded objective
   - s-shaped encoding function
   - over-sampling of low-probability states

Literature

psychophysics:
Weber's law, Fechner 1860, Thurstone '27...

encoding of stimuli:
Attneave '54, Barlow et al. '61, Laughlin '81...

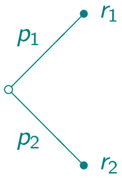
econ [riskless]:
Robson '01, Netzer '09, Rayo&Becker '07...

econ [risky, large noise]:
Friedman '89, Khaw&Li&Woodford '20, Frydman&Jin '19...

misspecification:
Berk '66, White '82, Esponda Pouzo '16, Heidhues et al. '18...

Table of Contents

Decision Problem



versus alternative s

risk-neutrality: lottery optimal $\Leftrightarrow r := \sum_i p_i r_i > s$

set of states and probabilities fixed, and DM observes s frictionlessly

the DM:

- measures each reward many times
- estimates the lottery value given the collected data
- controls the encoding function and sampling frequencies

Perception

perception strategy:

- encoding function $m : [\underline{r}, \overline{r}] \longrightarrow [\underline{m}, \overline{m}]$; exogenous span
- sampling frequencies $(\pi_i)_i \in \Delta$ (set of states)

DM samples signals $(i_k, \hat{m}_k)$, $k = 1, \ldots, n$:

- i_k specifies the state; sampling frequencies $\pi_i \neq p_i$
- $\hat{m}_k = m(r_{i_k}) + \varepsilon_k$; iid standard normal noise

DM is sophisticated: knows conditional signal distributions

decoding: a map from perception data to the estimate of the lottery

nearly complete information: $n \to \infty$

a posteriori optimal choice

Table of Contents

Simple Decoding

fix perception strategy $m(\cdot)$ and $(\pi_i)_i$

def simple decoding: DM's estimate of lottery value $= m^{-1}(\sum_{k=1}^{n} \hat{m}_k)$

Observation

The probability that the DM chooses the lottery in problem $(\mathbf{r}, s)$ converges a.s. to 1 (0) as $n \to \infty$ if

$$\sum_i \pi_i m(r_i) > (<)\, m(s).$$

EU maximizer with Bernoulli utility $m(\cdot)$ and subjective probabilities π_i

Oprea'23

two treatments:

1. genuine lottery $(p_i, r_i)_i$ vs safe option
2. certainty equivalent of $(p_i, r_i)_i$ vs safe option

nearly identical choices across the treatments

aggregation friction rather than risk aversion

our simple procedure fits Oprea's subjects

Maximum Likelihood Estimate

the DM is endowed with a compact set $\mathcal{A} \subseteq [\underline{r}, \overline{r}]^I$ of anticipated lotteries

forms ML estimate of the lottery

$$\mathbf{q}_n \in \arg\max_{\mathbf{r}' \in \mathcal{A}} \prod_{k=1}^{n} \varphi\left(\hat{m}_k - m\left(r'_{i_k}\right)\right)$$

Proposition

Suppose that the DM anticipates that the lottery involves no risk:

$$\mathcal{A} = \left\{\mathbf{r} \in [\underline{r}, \overline{r}]^I : r_i = r_j \text{ for all states } i, j\right\}.$$

Then, she follows the simple decoding procedure.

Proof

White '82: asymptotic MLE minimizes KL-divergence from the true data-generating process, among all anticipated processes

$$\text{MLE} \xrightarrow{\text{a.s.}} \arg\min_{r' \in \mathcal{A}} D_{KL}\left(f_{\mathbf{r}} \parallel f_{\mathbf{r}'}\right)$$

with Gaussian errors & no anticipated risk

$$D_{KL}\left(f_{\mathbf{r}} \parallel f_{\mathbf{r}'}\right) = \sum_{i=1}^{I} \pi_i \left(m\left(r_i\right) - m\left(r_i'\right)\right)^2$$

hence MLE of $m(r) \to \sum_{i=1}^{I} \pi_i m(r_i)$

Proof

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hence MLE of $m(r) \to \sum_{i=1}^{I} \pi_i m(r_i)$

Coarse Anticipation of Risk

DM anticipates lotteries to be measurable w.r.t. a partition of arms $\mathcal{K}$

Proposition

Prob that DM chooses the lottery in problem $(\mathbf{r}, s)$ converges to 1 [0] if

$$\sum_{J \in \mathcal{K}} p_J r_J^* > [<] \; s,$$

where, for each $J \in \mathcal{K}$,

- r_J^* is the certainty equivalent $m\left(r_J^*\right) = \sum_{i \in J} \frac{\pi_i}{\sum_{j \in J} \pi_j} m(r_i)$
- $p_J = \sum_{i \in J} p_i$ is the true probability of J

- anticipated risk: risk neutrality
- unanticipated risk: risk attitudes

Impact of Prior Information

let's bridge the gap between anticipated and unanticipated lotteries

joint limit of

- number of signals
- precision of prior density of Bayesian DM

effects of

- time pressure
- level of anticipated risk

Prior Belief and Sampling

prior $\propto \exp\left(-\frac{n}{\Delta}\sigma^2(\mathbf{r})\right)$ on $[\underline{r}, \overline{r}]^I$, where $\sigma^2(\mathbf{r}) = \sum_i p_i(r_i - r)^2$

DM samples $a \times n$ perturbed messages

- Δ – degree of the a priori anticipated risk
- a – attention span, sample size increases with a
- as n grows
  - sample size grows
  - risk becomes a priori unlikely

Lottery Perception

Proposition

The Bayesian estimate of lottery $\mathbf{r}$ converges to

$$\mathbf{q}^*(\mathbf{r}) = \underset{\mathbf{r}' \in [\underline{r}, \bar{r}]^I}{\arg\min} \left\{ \frac{1}{a\Delta} \sigma^2(\mathbf{r}') + \sum_i \pi_i \left(m\left(r_i\right) - m\left(r_i'\right)\right)^2 \right\}.$$

limiting cases

- $a\Delta$ large: close to risk-neutrality
- $a\Delta$ small: close to the simple procedure

unstable risk attitudes

- $a \to 0$ vs. $a \to \infty$: Kahneman's thinking fast/slow
- $\Delta \to 0$ vs. $\Delta \to \infty$: Rabin's paradox

Arrow-Pratt Measure

Proposition

Consider a lottery with small risk σ^2. The Bayesian estimate of the lottery value converges a.s. to

$$r + \frac{1}{2}\frac{m''(r)}{m'(r)} \cdot \sigma^2 \cdot \frac{1 + 4a\Delta m'^2(r)}{[1 + a\Delta m'^2(r)]^2} + o(\sigma^2).$$

- $a\Delta \to 0$: the usual Arrow-Pratt measure for $u(\cdot) = m(\cdot)$
- $a\Delta \to \infty$: risk-neutrality

Table of Contents

Objective

ex ante distribution of the decision problems $(\mathbf{r}, s)$

- all r_i iid from continuously differentiable density h
- s independently from continuously differentiable density h_s

ex ante minimization of

$$L(n) = \mathrm{E}\left[\max\{r, s\} - \mathbb{1}_{q_n > s} r - \mathbb{1}_{q_n \leq s} s\right]$$

- equivalent to maximization of the expected chosen reward

loss becomes tractable as n diverges

Limit Loss

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) = \text{const.}\,\mathrm{E}\left[\sum_i \frac{p_i^2}{\pi_i m'^2(r_i)} \mid r = s\right]\frac{1}{n} + O\left(\frac{1}{n^2}\right).$$

Limit Loss

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) \propto \mathrm{E}\left[\text{MSE conditional on tie}\right].$$

choice is distorted if s falls between r and value estimate q_n

condition on ties: small perception error distorts choice only if $r \approx s$

loss $\propto$ MSE

Limit Loss

Proposition

If the encoding function m is continuously differentiable, then

$$L(n) \propto \mathrm{E}\left[\sum_i p_i^2 \mathrm{MSE}(r_i) \text{ conditional on tie}\right].$$

MSE is a weighted sum of MSEs for each r_i

Limit Loss

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$\mathrm{MSE}(r_i)$ is mitigated by high π_i and $m'(r_i)$

Information-Processing Problem

$$\min_{m'(\cdot),(\pi_i)_i>0} \mathrm{E}\left[\sum_i \frac{p_i^2}{\pi_i m'^2(r_i)} \mid r = s\right]$$

$$\text{s.t.:} \quad \int_{\underline{r}}^{\overline{r}} m'(r)dr \leq \overline{m} - \underline{m}$$

$$\sum_i \pi_i = 1$$

attention allocation:

- high $m'(\tilde{r})$ focuses on the neighborhood of $\tilde{r}$
- high π_i focuses on the state i

constraints:

- $m(\cdot)$ is bounded – your scale can't be fine everywhere
- $\sum_i \pi_i = 1$ – you can't sample all the states frequently

Optimal Perception

suppose h and h_s are unimodal with a same mode and symmetric

Proposition

1. Optimal encoding function is s-shaped:
   $m(\cdot)$ is convex below and concave above the modal reward
2. Over-sampling of low-probability states:
   $\frac{\pi_J}{\pi_{J'}} > \frac{p_J}{p_{J'}}$ when $p_J < p_{J'}$

intuition:

1. focus on reward values that you're likely to encounter at ties
2. - over-sample states that you expect to be poorly informed on
   - you measure tail rewards poorly
   - conditional on tie, low-probability state has more spread-out rewards since $\sum_{J'} p_{J'} r_{J'} = s$ isn't too informative about r_J when p_J is small

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Conclusion

link between perception and risk attitudes arises when decoding is coarse

- informed comparative statics on perception predicts choice

optimality arguments get some stylized facts about perception right

- we introduce marginal reasoning to psychophsysics