Uncertainty, Investment and Productivity with Relational Contracts

Teaching Materials for JEEA Article

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Introduction

- **Paper:** about effect of uncertainty on investment and productivity with relational contracts
  - **issue:** how to reconcile evidence that effect is adverse with traditional theory that, with risk-neutral agents, would not be
- **Recent literature**: focussed on option value of not making *irreversible investments* (Dixit-Pindyck, 1994)
  - gives rise to *adjustment*, not *long-run equilibrium*, effect
    - calibration in Bloom (2009) has most of adjustment in 3 years
    - now that data $>$ 13 years after 2007 crisis, know not what happened
    - Bloom *et al* (2018) find need (unappealing) negative mean total factor productivity (TFP) shock to capture, not just increase in uncertainty
- **With relational contracts**: greater uncertainty with no change in mean affects long-run equilibrium investment with risk-neutral parties
  - *general investment* (equally valuable with alternative partners) reduced
  - *specific investment* (valuable only with current partner) may increase
    - of interest because Bloom (2014) comments that some investments increase with greater uncertainty

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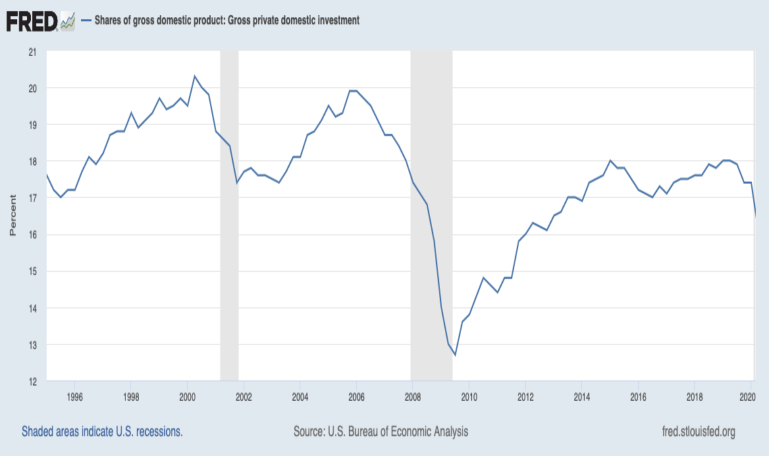
With each recession (shaded area), investment drops, then starts to grow but not back to previous path (as would with irreversible investment):



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This paper shows:

- **calibration** of relational contract model with risk-neutral parties and parameters based on Bloom *et al* (2018):
  - with *general investment* (equally valuable with alternative partners) can generate
    - *decrease* of magnitude in data with *just* greater uncertainty (no change in mean)
    - under a wide variety of conditions
  - with *specific investment* (valuable only with current partner) can do likewise
    - but under more restrictive conditions

The model: key elements

- **MacLeod & Malcomson (1989)** plus:
  - **uncertainty:** productivity of "effort" has *iid* shock each period
  - **investment in capital:** can enhance productivity of relationship
- **Principal's payoff** in period t conditional on being matched: $y(e_t, K, \theta_t) - W_t$, where W_t is payment to agent and:
  - $y(e_t, K, \theta_t)$: (non-contractible) output at t
  - $e_t \in [0, \bar{e}]$: agent's non-contractible effort at t, chosen after θ_t known
  - $K \in [0, \bar{K}]$: capital investment at start of relationship at cost $C(K)$
  - $\theta_t \in [\underline{\theta}, \bar{\theta}]$: *iid* random variable distributed $F(\theta, \sigma)$, with $dF(\theta, \sigma) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, parameterized by σ and observed by both parties at start of period t
- **Agent's payoff** in period t conditional on being matched: $W_t - c(e_t)$, where $c(e_t)$ is increasing and convex cost of effort
- **Payoffs if unmatched:** principal $\underline{v}(K, \sigma) \geq 0$, agent $\underline{u}(K, \sigma) \geq 0$, with $\underline{s}(K, \sigma) := \underline{u}(K, \sigma) + \underline{v}(K, \sigma) > 0$, for all $K \in [0, \bar{K}]$
- **Discount factor** for both parties δ

Key result on effort

Effort unenforceable in court because output and effort non-contractible

- so limited to what is in current interest of both parties
- $S(K,\sigma)$: joint (principal + agent) payoff from one period before shock θ realized given K and σ

Proposition

An effort schedule $e(K,\theta,\sigma)$ that generates expected joint payoff $S(K,\sigma)$ each period with capital stock K and distribution σ can be implemented by a stationary contract if and only if

$$\frac{\delta}{1-\delta}\left[S(K,\sigma)-\underline{s}(K,\sigma)\right] \geq c(e(K,\theta,\sigma)), \quad \text{for all } \theta \in \left[\underline{\theta},\bar{\theta}\right]. \tag{1}$$

- (1) requires joint payoff *gain* from future exceeds cost of effort now
  - irrelevant how $S(K,\sigma)$ divided between principal and agent

Intuition for key result on effort

- **Intuition:** if agent not going to be paid today for effort today:
  - will not deliver more effort than compensated for by gain from future continuation of relationship
  - so: maximum effort constrained by total future gain from continuation
- **Not necessary** that agent receives all this future payoff gain
  - could receive bonus pay from principal today
  - but: only in principal's interest to pay bonus if less than future payoff gain from continuing relationship
- **So:** how total future payoff gain from continuation divided between principal and agent unimportant for how much effort can be achieved

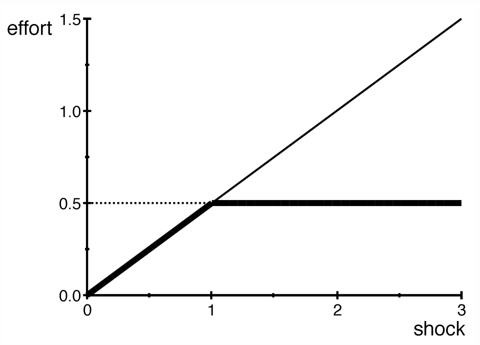
Implications of key result on effort

Key equation is

$$\frac{\delta}{1-\delta}\left[S(K,\sigma) - \underline{s}(K,\sigma)\right] \geq c(e(K,\theta,\sigma)), \qquad \text{for all } \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \qquad (2)$$

- With *iid* shocks, left-hand side is independent of current θ
  - first-best effort $e^*(K,\theta)$ is increasing in θ
  - if constraint not binding for θ, implement first-best effort
  - so: if constraint binding for $\theta = \tilde{\theta}$, it is certainly binding for all higher θ
- Implication: binding constraint restricts how much can adjust to θ
  - if, without constraint, joint payoff is linear in θ (risk neutrality)
  - with constraint, joint payoff is strictly concave
    - so: make general investment choice as if risk averse
  - specific investment relaxes constraint (2) because increases joint payoff gain from future
    - so greater uncertainty may increase return to investment

Illustration of effort constraint



- Thin line: first-best effort for given capital stock
- Dotted line: highest effort sustainable given total future payoff gain
- Thick line: optimal effort with relational contract
- Use term *cutoff shock* for shock above which effort with relational contract constrained
  - equals 1 in case illustrated in figure

Implications of effort constraint

- Effort constraint makes risk-neutral parties behave as if risk-averse
- Potential explanation for various puzzles
  - example: why private sector may not undertake socially worthwhile major infrastructure projects
- In this paper, apply to puzzle of response of investment to economic shocks
  - much of paper taken up with propositions for general specification showing effect of less risky distribution (in sense of second-order stochastic dominance) on general and on specific investment
  - here will concentrate on showing empirical implications in calibrated version of model

Functional forms for calibration

- As literature, Cobb-Douglas production and iso-elastic cost functions:

$$
\begin{aligned}
y(e, K, \theta) &= \theta^{\gamma} K^{\alpha} e^{\beta}, \quad \alpha, \beta, \gamma > 0, \quad \alpha + \beta \leq 1; \\
c(e) &= ce^{n}, \quad c > 0, n \geq 1, n > \beta/(1-\alpha); \\
C(K, \sigma) &= C(\sigma) K^{k}, \quad C(\sigma) > 0, k \geq 1.
\end{aligned}
$$

- Joint payoff if no relational contract constraint (first-best effort)

$$
s(e^{*}(K, \theta), K, \theta) = \left(1 - \frac{\beta}{n}\right) \left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}} \theta^{\frac{\gamma}{1-\beta/n}} K^{\frac{\alpha}{1-\beta/n}}
$$

- Risk neutrality requires $\gamma = 1 - \beta/n$:
  - so expected joint payoff affected by θ only through its mean
- Distribution of θ log-normal:
  - implies always *interior cutoff shock* for first-best effort

Implications of functional forms for relational contract

- Joint payoff with relational contract and $\tilde{\theta}$ cutoff shock

$$s\left(e^*\left(K,\tilde{\theta}\right),K,\theta\right)=\left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}}\tilde{\theta}^{\frac{\gamma}{1-\beta/n}}K^{\frac{\alpha}{1-\beta/n}}\left[\left(\frac{\theta}{\tilde{\theta}}\right)^{\gamma}-\frac{\beta}{n}\right] \quad (3)$$

Note that strictly concave in θ for given $\tilde{\theta}$ when $\gamma < 1$

- A **proposition** gives conditions for change from σ_L (low risk) to σ_H (high risk) with relational contract to exactly match
  - ratios of productivity and capital of change from σ_L' to σ_H' without relational contract
    - for *general capital*, quite generally exists a relational contract specification that does this
    - for *specific capital*, less flexibility

Goal of calibration exercise

- To generate empirically more realistic simulations, Bloom *et al* (2018) use 2% negative first-moment shock
  - implies $E\left(\theta \mid \sigma'_H\right) / E\left(\theta \mid \sigma'_L\right) = 0.98$
- Here look for relational contract specifications with $E\left(\theta \mid \sigma_H\right) = E\left(\theta \mid \sigma_L\right) \equiv E\left(\theta\right)$
  - with same effect on capital and productivity
- To explore, calibrate model with parameters from Bloom *et al* (2018)
- Convenient to express results in terms of $\hat{\theta}^i\left(\sigma\right)$, for $i = G, S$
  - defined as optimal cutoff θ at which relational contract constraint becomes binding for general and specific capital respectively when capital chosen optimally
  - and in terms of

$$\hat{S}\left(\sigma\right) = \frac{\delta}{1-\delta}\left[S\left(K,\sigma\right) - \underline{s}\left(K,\sigma\right)\right]. \tag{4}$$

Parameters for calibration

Based on Bloom *et al* (2018), combining aggregate σ^A and firm σ^Z shocks

Parameter	Value	Source
δ	$0.95^{1/4}$	Bloom *et al* (2018), annual discount factor of 95%
α	0.25	Factor share with isoelastic demand, 33% markup
β	0.5	As α with labour share 2/3, capital share 1/3
n	1	Implied by Bloom *et al* (2018) model
k	1	Implied by Bloom *et al* (2018) model
σ_L^A	0.67	Bloom *et al* (2018) estimate, %
σ_H^A/σ_L^A	1.6	Bloom *et al* (2018) estimate
σ_L^Z	5.1	Bloom *et al* (2018) estimate, %
σ_H^Z/σ_L^Z	4.1	Bloom *et al* (2018) estimate
σ_L	0.10	Calculated combined σ_L^A and σ_L^Z for θ
σ_H/σ_L	4.07	Calculated from combined σ_H^A and σ_H^Z for θ

Table: Parameter values for calibration

Calibration results for general capital

- For *general capital*, continuum of values for $\hat{\theta}^G(\sigma_L)/E(\theta)$ between 0 and 1.57507
  - that match ratios of productivity and capital change with $E(\theta \mid \sigma_H) = E(\theta \mid \sigma_L) \equiv E(\theta)$
  - thus values of $\hat{\theta}^G(\sigma_L)$ both below and above the mean
- Columns in next table illustrate with values of $\hat{\theta}^G(\sigma_L)/E(\theta)$ interspersed between these
- In each case, $\hat{\theta}^G(\sigma_H)/E(\theta)$ is at least as high as $\hat{\theta}^G(\sigma_L)/E(\theta)$
  - so higher cutoff θ below which effort is first best for σ_H than for σ_L
  - difference greatest for $\hat{\theta}^G(\sigma_L)$ somewhat above its mean
  - difference essentially negligible for $\hat{\theta}^G(\sigma_L)$ at each end of its acceptable range

General capital matching specifications

	Matching specifications						
$\frac{\hat{\theta}^G(\sigma_L)}{E(\theta)}$	0.01	0.25	0.50	0.75	1.00	1.25	1.57507
$\frac{\hat{\theta}^G(\sigma_H)}{E(\theta)}$	0.01008	0.2502	0.504	0.809	1.312	1.570	1.57507
$\frac{\hat{S}(\sigma_H)C(\sigma_H)}{\hat{S}(\sigma_L)C(\sigma_L)}$	0.98078	0.9808	0.988	1.057	1.286	1.231	0.980
$\frac{\hat{S}(\sigma_H)}{\hat{S}(\sigma_L)}$ [a]	0.9207	0.9207	0.927	0.992	1.207	1.805	0.920

Table: General capital relational contract values with $E(\theta \mid \sigma_H) = E(\theta \mid \sigma_L)$ matching $E(\theta \mid \sigma'_H) / E(\theta \mid \sigma'_L) = 0.98$ (Note:[a] for $C(\sigma_L) / C(\sigma_H) = 0.939$.)

- $\hat{S}(\sigma_H) / \hat{S}(\sigma_L)$ is ratio of joint payoff gain from continuing relationship over ending it for σ_H to that for σ_L
  - 3rd row gives the ratio of $\hat{S}(\sigma) C(\sigma)$ for $\sigma = \sigma_H$ to that for $\sigma = \sigma_L$
  - Bloom *et al* (2018) do not report capital costs corresponding to $C(\sigma)$
  - could infer $C(\sigma_L) / C(\sigma_H) = 0.939$ from long-run effects on capital and productivity but limitations

General capital matching specifications: implications

- Consider two kinds of shocks
  - *systemic*: affects values of both continuing and ending relationship
    - for systemic shocks, $\hat{S}(\sigma)$ independent of σ
  - *specific*: affects only value of continuing relationship
- If shocks entirely *systemic*, $\hat{S}(\sigma_H) = \hat{S}(\sigma_L)$
  - implies $\hat{\theta}^G(\sigma_L)/E(\theta)$ between 0.75 and 1 or near highest value
- If shocks entirely *specific*, $\hat{S}(\sigma_H) < \hat{S}(\sigma_L)$
  - because lower joint payoff to continuing relationship from adverse impact on capital but no impact on joint payoff to separating
  - bottom row of table indicates $\hat{\theta}^G(\sigma_L)$ further from the mean than with purely systemic shocks
  - still consistent with model and calculated $C(\sigma_L)/C(\sigma_H)$ as long as $\hat{S}(\sigma_H)/\hat{S}(\sigma_L) \geq 0.92$
    - 8% reduction in joint gain from continuing relationship over ending it

Specific capital matching specifications

Some evidence of at least some capital specificity

- Same pairs of cutoff values of θ apply as for general capital
- Table below gives single pair that satisfies other conditions
- For change in risk that entirely specific, $\underline{s}(\sigma_H) = \underline{s}(\sigma_L)$
- If some risk systemic, might expect $\underline{s}(\sigma_H) < \underline{s}(\sigma_L)$
  - when higher risk adversely affects payoffs if relationship ends
  - for $C(\sigma_L)/C(\sigma_H) = 0.939$, ratio in table is $\underline{s}(\sigma_H)/\underline{s}(\sigma_L) = 0.94$
    - consistent with some risk being specific

$\hat{\theta}^S(\sigma_L)/E(\theta)$	$\hat{\theta}^S(\sigma_H)/E(\theta)$	$\frac{\underline{s}(\sigma_H)C(\sigma_H)}{\underline{s}(\sigma_L)C(\sigma_L)}$	$\underline{s}(\sigma_H)/\underline{s}(\sigma_L)^{a}$
0.93	1.15	0.97	0.94

Table: Specific capital relational contract values with $E(\theta \mid \sigma_H) = E(\theta \mid \sigma_L)$ matching $E(\theta \mid \sigma'_H)/E(\theta \mid \sigma'_L) = 0.98$ (Note: [a] for $C(\sigma_L)/C(\sigma_H) = 0.939$.)

Comments on calibration results

- *Calibration design*: to match *long-run equilibrium change* in capital and productivity
  - no dynamics, so not designed to match adjustment path
  - for that could use probabilistic switching between σ_L and σ_H regimes
    - but tricky with relational contract model
- *Capital rigidity*: model allows no adjustment of capital in response to shocks
  - either upwards or downwards
  - one-sided irreversibility tricky to handle with relational contracts

Conclusions on relational contract model

Theoretical effects of greater uncertainty

- **general investment:** risk-neutral parties choose as if risk averse when rely on relational contract
  - so: greater uncertainty reduces general investment for same mean
- **specific investment:** relaxes relational contract constraint
  - greater uncertainty for same mean may increase specific investment because relaxing constraint becomes more valuable

Calibrated effects of greater uncertainty

with functions and parameters based on Bloom *et al* (2018):

- **general investment:** can capture measured falls in capital and productivity without (unappealing) negative first-moment shock under wide variety of conditions
- **specific investment:** can do likewise but under more restrictive conditions