# Hard-to-Interpret Signals

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# Background

- Cornerstones of Bayesian theory: subjective prior and Bayesian updating
- Ellsberg: limitations of the single prior – no role for confidence/ambiguity.
  - An intuitive thought experiment, stimulated many laboratory experiments

# Goal

- Seek parallel critique of updating component, centered on uncertainty (or incomplete confidence) about how to interpret a signal
  - We offer definition, a thought experiment and a laboratory implementation
- Two distinct kinds of ambiguity:
  - Uncertainty about prior probabilities VERSUS uncertainty about posteriors or likelihood function
  - **prior ambiguity** vs **signal ambiguity**

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**Leading example:** a policy-maker faces uncertain serious health outcomes due to COVID-19:

- New virus: uncertainty about the risk that an infected individual will suffer serious health outcomes and about the infection rate.
- Signals: case-fatality-rates (# of deaths of confirmed cases / # of confirmed positive cases) in different locations.
- Unknown: how many individuals have been infected but not confirmed (or died without being confirmed positive cases).
- How would a policy maker react to this information when taking very costly actions?

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- Here: a signal may provide weak support for both an event and its complement, **contrary to the martingale property of Bayesian updating.**
  Prior prob of an event need not be an average (lie in the convex hull) of posteriors because signal is hard-to-interpret

# Environment

- Payoff urn (ambiguous): $R + B = 10, \;\; R, B \geq 1$
- All bets to be considered pay 100 or 0.
- All bets will be on the color of the ball ($s \in \{R, B\}$) drawn from this urn
- Consider two *ordered* scenarios

# Choice Problems

- **Unconditional choice**: probability equivalents to the bets

$$f_s \sim_0 \left(100, p_{0,s}\right), \; s = R, B$$

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- **Conditional choice**: The DM is informed that there is a second "signal urn" that is constructed by *adding* an equal number $N$ of red and black balls to the payoff urn, where $N$ is not completely specified. Probability equivalents for the bets on red and black *from the payoff urn conditional on each possible draw* $\sigma \in \boldsymbol{\Sigma} = \{\sigma_R, \sigma_B\}$ from the signal urn:
For each color $s = R, B$ and signal $\sigma = \sigma_R, \sigma_B$,

$$f_s \sim_\sigma \left(100, p_{\sigma,s}\right) = P_{\sigma,s}$$

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- Reason for ordered scenarios – signals not even anticipated at unconditional stage – hence **isolate processing of hard-to-interpret signals from ex ante ambiguity.** NOT dynamic choice. Rather, choice in two different but related settings
- Alternatively, if the DM is myopic, there is no difference between the ordered and dynamic scenario. The problem is that myopic behavior is difficult to observe.

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- Assume $p_{0,R} = p_{0,B}$, $p_{\sigma_R,R} = p_{\sigma_B,B}$, $p_{\sigma_R,B} = p_{\sigma_B,R}$

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  Behavior indicates that "signals increase ambiguity"
- Affinity and indifference defined in obvious way

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- Intuitive/plausible that aversion to uncertainty about signal interpretation (posteriors) gives $p_{\sigma_R,R} < .67$ and $p_{\sigma_B,R} < .33$

$$p_{0,R} = \frac{1}{2} > \frac{1}{2} p_{\sigma_R,R} + \frac{1}{2} p_{\sigma_B,R}$$

# Definition

$\Sigma = \{\sigma_R, \sigma_B\}$ : noisy signals
$\alpha_{\sigma_R} + \alpha_{\sigma_B} = 1$ respective probabilities of signals.

**Definition (aversion to signal ambiguity)**

$\exists \quad 0 \leq \alpha_{\sigma_R} \leq 1$ s.t.

$$\begin{aligned} p_{0,R} &> \alpha_{\sigma_R} p_{\sigma_R,R} + (1 - \alpha_{\sigma_R})\, p_{\sigma_B,R} \text{ and} \\ p_{0,B} &> \alpha_{\sigma_R} p_{\sigma_R,B} + (1 - \alpha_{\sigma_R})\, p_{\sigma_B,B} \end{aligned}$$

Let $(p_{\sigma_R,R} - p_{\sigma_B,R}) > 0$. With symmetry implies **diversity**

$$(p_{\sigma_R,R} - p_{\sigma_B,R}) \cdot (p_{\sigma_R,B} - p_{\sigma_B,B}) < 0$$

# Definition (cont.)

**Lemma**

*The DM is averse to signal ambiguity iff*

$$p_{0,R} > \frac{1}{2} p_{\sigma_R,R} + \frac{1}{2} p_{\sigma_B,R}$$

# Definition - qualifier

We require $\exists$. What about $\forall$?

$$\Longrightarrow \text{(with symmetry) } p_{0,R} > p_{\sigma_R,R} \text{ and } p_{0,R} > p_{\sigma_B,R},$$

which is very strong and is still weaker that dilation (Good, 1974; Walley, 1991) – in maxmin: sets of posteriors enlarge the set of priors for every signal realization. (rejected by Shishkin and Ortoleva, 2023).

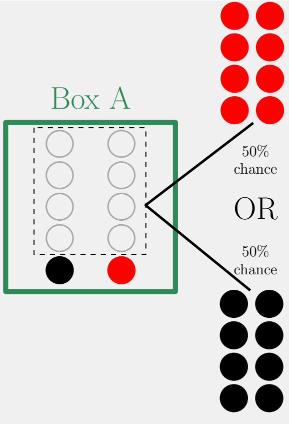
Our view: a signal may carry informational content, but may increase uncertainty. Our proposed definition weighs the two componsnets.

$\Longrightarrow$A model that is inconsistent with our definition, precludes aversion to signal ambiguity.

# Experimental design

- Between-subject design that builds on the numerical example to measure sensitivity to signal ambiguity, **compared** to updating when signal preciseness in known.
- Payoff urn: 9R1B or 1R9B, each with probability .5
  - If subjects reduce compound lotteries, $p_{0,R} = p_{0,B} = .5$
  - Extensive evidence that many subjects do not reduce compound lotteries, and that they treat compound risk similarly to ambiguity. Halevy (ECMA 2007) and especially Chew, Miao and Zhong (ECMA 2017) document that attitude towards two-point ambiguity and compound-risk (as we have here) are significantly correlated.
  - Can measure association between ROCL and Bayesian updating.

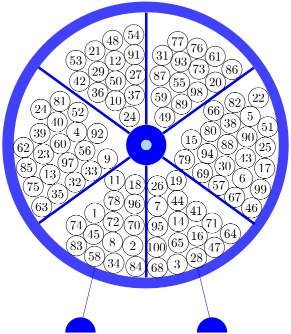
# Payoff urn




Use of objective lottery justifies symmetry. Subject chooses a color (R or B) to bet on (excludes hedging).

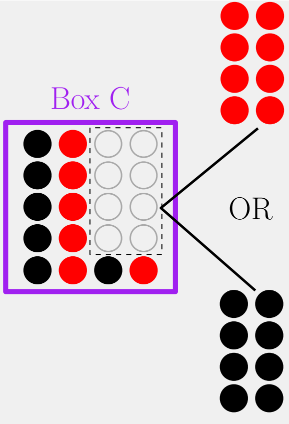
# Elicitation of probability equivalents

Basket B



Through a 2-stage calibration to an objective urn using choice list.

# Signal urn - risk control



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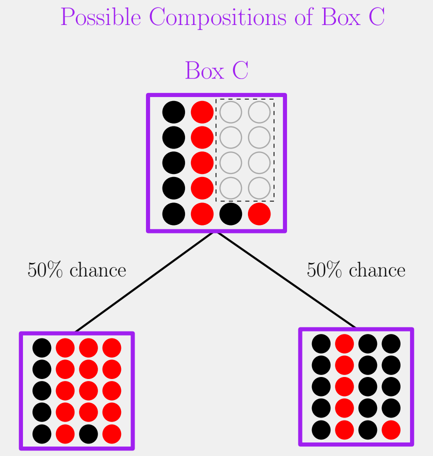
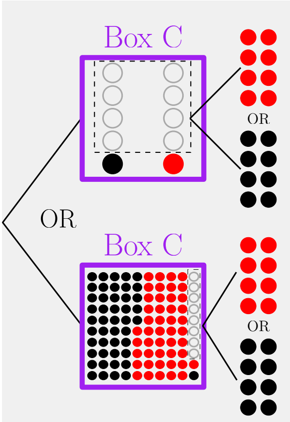



$p_{\sigma_R,R} = 0.66$

# Signal urn - ambiguity



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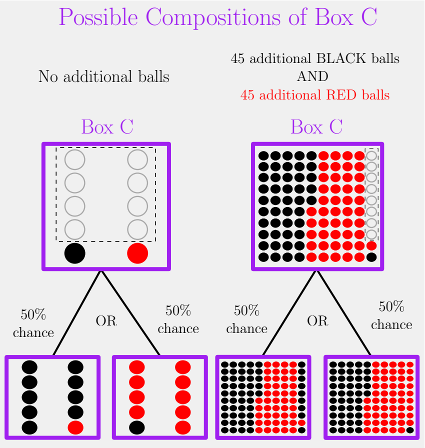
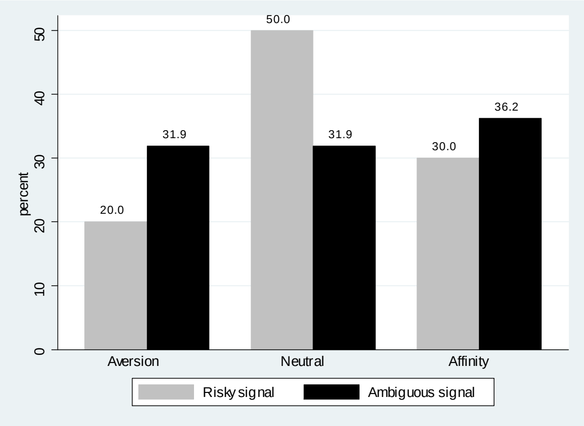



$$p_{\sigma_R, R} = 0.82 \text{ or } p_{\sigma_B, R} = 0.532$$

# Elicitation

- Subject chooses a color to bet on (assume red), then using basket B:
  - $p_{0,R}$ – elicited before subject was told anything about a signal urn.
  - $p_{\sigma_R,R}$
  - $p_{\sigma_B,R}$
- RIS: hedging works against finding effect of ambiguity as subjects may hedge between $\sigma_R$ and $\sigma_B$.
- Treatment effect: distribution of $(0.5p_{\sigma_R,R} + 0.5p_{\sigma_R,R}) - p_{0,R}$ in risk vs. ambiguity.
- Prize: $20
- 154 subjects: Risk - 68, Ambiguity - 86.
- No multiple switching and updating consistent with information: 129 subjects (84%): Risk - 50, Ambiguity - 69

# Updating and attitude to signal ambiguity




One sided proportional test 0.018, Fisher exact 0.028.

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- Association between ROCL and Bayesian updaing (in the risky signal control)
- Association between ROCL (and indirectly – prior ambiguity) and attitude to signal ambiguity (in the ambiguous signal treatment)
- Strong associations in both conditions ($p$-value of Fisher exact test is 0.004 in both)

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2. Non-Bayesian unconditional choice (Ellsbergian) with Bayesian updating
3. Fully non-Bayesian (Maxmin):
   multiple priors $\Longrightarrow$ Ellsbergian ambiguity, affinity to signal ambiguity.
   multiple likelihoods (with different updating rules) $\Longrightarrow$ aversion to signal ambiguity

# Related Literature

- Experimental
  - Liang (2023): updating ambiguous belief using risky signal and ambiguous information with risky belief
  - Shishkin and Ortoleva (2023): ambiguous information and dialation.
- Decision Theoretic:
  - Updating under ambiguity (Gilboa-Schmeidler 1993; Pires 2002)
  - "Ambiguous signals" (Epstein-Schneider 2007,8) - we build on their thought experiment
  - Imprecise information, processing and preference (Gajdos et al 2008)

# Related Literature

- Applications
  - Ambiguous communication can be optimal:
    mechanism design (Bose-Renou 2014); cheap talk (Kellner-Le Quement 2017,8 and Kellner, Le Quement and Riener 2022)
    sender-receiver games (Blume-Board 2014)
    ambiguous persuasion (Beauchene, J. Li, M. Li 2017)
    Voting (Fabrizi, 2019)
  - Finance (Epstein-Schneider, 2008, 2010; Hansen-Sargent; Ju and Miao 2012)

# Concluding remarks - theory

- The ambiguity literature has typically focussed on the prior stage and not on the nature of, or response to, signals. That has led to predominance of models like multiple-priors combined with single likelihoods
  - One should be aware of the behavioral meaning of such specifications

# Concluding remarks - theory

- Machina-Schmeidler (1992,1995) separate "probabilistic beliefs" from the SEU functional form axiomatically – *probabilistic sophistication (PS)*. It provides a natural benchmark (neutrality) for Ellsberg-style ambiguity.
  - We identify "Bayesian-like" updating as the benchmark for modeling attitude towards signal ambiguity, and we do so in a general nonparametric preference framework. Key is the "martingale property," extended here beyond models with probabilistic beliefs

# Concluding remarks - empirical

- Updating is difficult and most subjects do not perform Bayesian updating even for a risky signal.
- When signals are hard-to-interpret: significantly higher deviations from the Bayesian benchmark, and associated with prior ambiguity aversion.
- Suggest that for subjects in the lab $\Longrightarrow$ signals generate ambiguity