Aggregate Properties of Open Economy Models with Expanding Varieties

Saroj Bhattarai[1]   Konstantin Kucheryavyy[2]

[1]University of Texas at Austin

[2]University of Tokyo

June, 2023

Motivation and Research Questions

International Real Business Cycle

- IRBC model of Backus *et al* (1994)
  - Perfect competition and representative firms
  - Transmission mechanisms well-understood
  - Difficulty matching aggregate data moments
    - International corr. of output, consumption, investment, hours
    - Volatility of trade balance and cyclicality of real exchange rate
- Dynamic versions of Krugman (1980) and Melitz (2003)
  - Monopolistic competition, heterogeneous firms, and endogenous entry/exit
  - **Theoretical connections to IRBC?**
  - **Transmission mechanisms?**
    - No satisfying answer in the literature
  - **Ability to match aggregate data moments?**
    - *Quantitatively* similar to IRBC (**But why?**)

This Paper: Theoretical Connections to IRBC

External Economies of Scale

- **Unified model:** IRBC with EES
  - EES in production of intermediate and final goods
- Mapping from Krugman/Melitz models to the unified model

$$\left.\begin{array}{l}\textit{Love-of-variety effect}\\ \text{in the Krugman model}\\[6pt] \textit{Selection effect}\\ \text{in the Melitz model}\end{array}\right\} \Rightarrow \text{ EES in the unified model}$$

- But mapping $\Rightarrow$ tight restrictions on params. in the corresponding unified model

Unified	α_X	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	$\sigma - 1$
Krugman	$\frac{1}{\sigma^K}$	$\frac{1}{\sigma^K - 1} - \frac{1}{\sigma^K}$	$\frac{1}{\sigma^K}$	0	$\sigma^K - 1$
Melitz	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M \theta^M}$	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$	θ^M

This Paper: From Theory to Quantitative Results

- **More theory:**
  - *Generalized* Krugman/Melitz models relax restrictions
    - $\Rightarrow$ Isomorphism with the unified model
    - $\Rightarrow$ **Explanation of transmission mechanisms**
- **Quantitative results:**
  - Standard versions of Krugman/Melitz models are quantitatively similar to IRBC
    - **Standard calibrations $\Rightarrow$ positive and small externalities with tight relationships between them**
  - **Need negative and large capital externality to improve fit**

Related Literature

- IRBC models
  - Backus *et al* (1994), Heathcote and Perri (2002)
- International business cycle models with firm entry and selection into exporting
  - Ghironi and Melitz (2005), Alessandria and Choi (2007), Fattal Jaef and Lopez (2014)
- Isomorphisms between static Krugman, Melitz, and perfect competition models of trade with EES
  - Kucheryavyy *et al* (2023)

Standard IRBC: Summary

- Time is discrete and horizon is infinite
- Four production sectors:
  - Intermediate, final aggregate, consumption, and investment
- Capital and labor as primary factors of production
  - Used only in intermediate sector
- Intermediate goods are the only traded goods
  - Iceberg trade costs
  - Used in final good sector
- Consumption and investment one-to-one from final good
- Shock: only aggregate productivity in intermediate sector
- Perfectly competitive product markets (Armington)

Standard IRBC: Production Technologies and Households

- Output of intermediates in country n :

$$X_{nt} = Z_{x,nt} K_{x,nt}^{\alpha_x} L_{x,nt}^{1-\alpha_x},$$

$Z_{x,nt}$ is exogenous shock, $K_{x,nt}$ capital, $L_{x,nt}$ labor

- Output of final good:

$$Y_{nt} = \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

$X_{ni,t}$ is amount of intermediate good country n buys from i

- Standard household problem Household Problem
  - Law of motion of capital: $K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}$
  - Endogenous labor supply, L_{nt}

Standard IRBC Calibration

- Follow Heathcote and Perri (2002)
- Per-period utility (discount factor β)

$$U(C_{nt}, L_{nt}) = \frac{1}{1-\gamma}\left[C_{nt}^{\mu}(1-L_{nt})^{1-\mu}\right]^{1-\gamma}$$

- Productivity process for intermediates

$$\begin{bmatrix}\log(Z_{x,1t})\\ \log(Z_{x,2t})\end{bmatrix} = \begin{bmatrix}0.97 & 0.0\\ 0.0 & 0.97\end{bmatrix} \times \begin{bmatrix}\log(Z_{x,1,t-1})\\ \log(Z_{x,2,t-1})\end{bmatrix} + \begin{bmatrix}\varepsilon_{x,1t}\\ \varepsilon_{x,2t}\end{bmatrix},$$

  with

$$\begin{bmatrix}\varepsilon_{x,1t}\\ \varepsilon_{x,2t}\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}0\\ 0\end{bmatrix}, \begin{bmatrix}0.0073^2 & 0.0\\ 0.0 & 0.0073^2\end{bmatrix}\right)$$

- Values of parameters:

β	γ	μ	α_x	δ	τ_{ni}	σ
0.99	2	0.34	0.36	0.025	5.67	2

  - τ_{ni} is iceberg trade cost

Results for Final Good Shock

Standard IRBC: Moments

Moment	Data	IRBC Complete	IRBC Bond	IRBC Fin. aut.
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	−0.03	0.02	0.15
Corr (C_1, C_2)	0.36	0.47	0.11	0.17
Corr (I_1, I_2)	0.30	−0.39	−0.35	0.14
Corr (L_1, L_2)	0.42	−0.30	−0.04	0.13
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.49	−0.60	
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.61	0.50	0.65

Notes: Data moments are from Heathcote and Perri (2002), Table 2. Calibration follows Heathcote and Perri (2002). All series have been HP-filtered. $\mathrm{GDP}_n = (W_n L_n + R_n K_n)/P_{Y,nt}$, $\mathrm{TB}_1 = P_{X,1}X_1/P_{Y,1} - Y_1$, $\mathrm{ReR} = P_{Y,2}/P_{Y,1}$.

Focus on complete markets for the rest of the talk

Dynamic Version of Standard Melitz Model

- Monopolistic competition in intermediate good sector
- Continuum of varieties produced in each country
  - Pareto distribution of efficiencies of production with shape θ^{M}
  - Elasticity of substitution between varieties σ^{M}
  - Each country n produces a unique set of varieties Ω_{nt}
    - M_{nt} is the measure
- Production technology of variety $\nu \in \Omega_{nt}$:

$$x_{nt}(\nu) = Z_{x,nt} z_n(\nu) l_{nt}(\nu)$$

  - $l_{nt}(\nu)$ is labor, $z_n(\nu)$ is efficiency, and $Z_{x,nt}$ is shock
  - **Note:** Only labor used in production

Dynamic Version of Standard Melitz Model

- Sunk costs of entry into the economy (free entry)
- Paid in terms of labor
  - Sunk cost equal to $W_{nt}/Z_{x,nt}$
- Fixed costs of serving markets:
  - Paid for all markets in terms of **destination** country labor
- Dynamics comes from law of motion of varieties

$$M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}$$

  - $M_{I,nt}$ is the number of varieties entering country n in period t
  - δ is exogenous exit rate of varieties

Dynamic Version of Standard Melitz Model

This model maps into standard IRBC with

- Number of firms/varieties corresponds to capital
- External economies of scale in interm. and final good sectors
- But investment in terms of **labor** instead of **final good**
- Additional shocks perfectly correlated with interm. sector:
  - In final aggregate and investment sectors

Mapping of Melitz Model into IRBC

- Output of intermediate good:

$$X_{nt} = S_{X,nt} K_{X,nt}^{\alpha_X} L_{X,nt}^{1-\alpha_X},$$

$S_{X,nt} \equiv Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$ is taken by producers as given

$$\boxed{\alpha_X = \frac{\sigma^M - 1}{\sigma^M \theta^M}, \quad \psi_{X,K} = \frac{1}{\sigma^M \theta^M}, \quad \psi_{X,L} = \frac{\sigma^M - 1}{\sigma^M \theta^M}}$$

- Output of final good:

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

$S_{Y,nt} \equiv [Z_{X,nt}]^{\psi_Y} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}$ is taken by producers as given,

$$\boxed{\psi_Y = \frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}, \quad \sigma \neq \sigma^M}$$

Final good in Melitz model

- Investment: $I_{nt} = Z_{X,nt} L_{I,nt}$ (in IRBC: $I_{nt} = Y_{I,nt}$)

Standard Melitz Model: Moments

- Use $\sigma^{M} = 3.8$ and $\theta^{M} = 3$, and other parameters as in IRBC
  - In particular, trade elasticity is $\sigma - 1 = 1$

Moment	Data	IRBC	Mel
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	−0.03	−0.09
Corr (C_1, C_2)	0.36	0.47	0.45
Corr (I_1, I_2)	0.30	−0.39	−0.26
Corr (L_1, L_2)	0.42	−0.30	−0.46
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.49	0.61
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.61	0.67

- **Similar performance of Melitz model vs IRBC**
- Additionally, remove differences other than externalities:
  - Investment in terms of labor and additional shocks

Melitz Model: Moments

Investment in terms of the final good and no additional shocks

Moment	Data	IRBC	Mel	Mel Inv. final
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	−0.03	−0.09	−0.11
Corr (C_1, C_2)	0.36	0.47	0.45	0.37
Corr (I_1, I_2)	0.30	−0.39	−0.26	−0.44
Corr (L_1, L_2)	0.42	−0.30	−0.46	−0.43
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.49	0.61	−0.20
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.61	0.67	0.69

Still no better than IRBC. Why?

IRBC Investment Labor

Performance of Melitz vs. IRBC

- Melitz model with final good inv. and no additional shocks
  - Difference from IRBC is three additional scale elasticities:

Model	α_X	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y
IRBC	α_X	0	0	0
Melitz	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M \theta^M}$	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$

$$X_{nt} = \left[Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}} \right] K_{X,nt}^{\alpha_X} L_{X,nt}^{1-\alpha_X}, \quad Y_{nt} = \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y} \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- Implied calibrations [for $\sigma^M = 3.8$ and $\theta^M = 3$]:

Model	α_X	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y
IRBC	0.36	0	0	0
Melitz	0.25	0.088	0.25	0.024

- **Implied scale elasticities are small to make difference**

Generalization of IRBC

- Melitz model has tight relationship between α_X, $\psi_{X,K}$, $\psi_{X,L}$, ψ_Y
- Go to IRBC with unrestricted parameters to explore roles of $\psi_{X,K}$, $\psi_{X,L}$, ψ_Y independently
  - "Unified model" in the paper
  - Can be micro-founded through a generalized Melitz model
    - Proposition 1 in the paper
- **Main questions:**
  - **How do different scale elasticities affect the transmission mechanism of a productivity shock?**
  - **What scale elasticities give the best fit with data moments?**

Role of External Economies of Scale

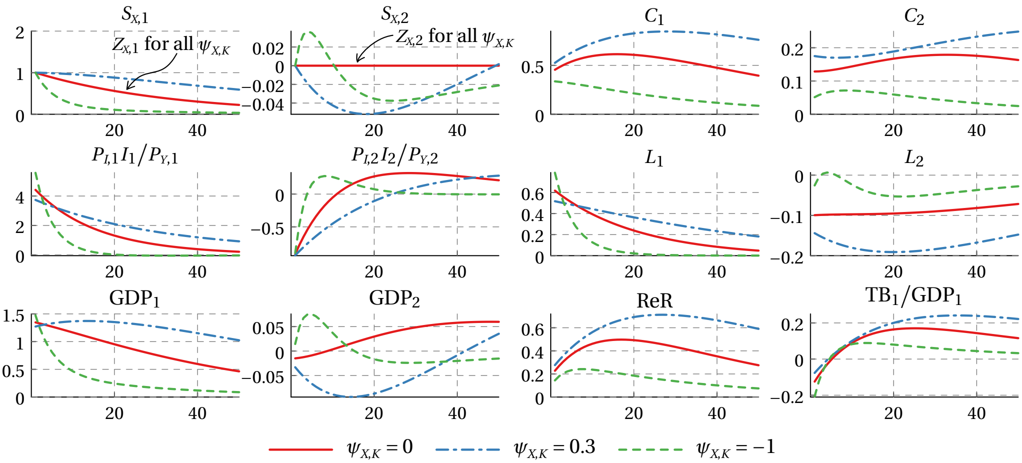
Moment	Data	IRBC	$\psi_{X,K}$		$\psi_{X,L}$		ψ_{Y}	
			0.3	-1	0.7	-1	0.2	-1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	−0.03	−0.07	0.08↑	−0.17	0.10	−0.31	0.12
Corr (C_1, C_2)	0.36	0.47	0.55	0.34↓	0.25	0.62	0.19	0.74
Corr (I_1, I_2)	0.30	−0.39	−0.47	−0.26↑	−0.48	−0.31	−0.70	0.01
Corr (L_1, L_2)	0.42	−0.30	−0.52	0.00↑	−0.35	−0.25	−0.52	−0.30
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.49	−0.40	−0.57	−0.55	−0.45	−0.66	0.62
Corr $\left(\mathrm{ReR}, \frac{\mathrm{GDP}_1}{P_{Y,1}}\right)$	0.13	0.61	0.67	0.46↓	0.64	0.58	0.68	0.44

Notes: Arrow means moment improves relative to IRBC (in the shown direction).

$$X_{nt} = \left[Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right] K_{X,nt}^{\alpha_X} L_{X,nt}^{1-\alpha_X}, \quad Y_{nt} = \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Role of Capital Externalities

Responses to a 1% home intermediate good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

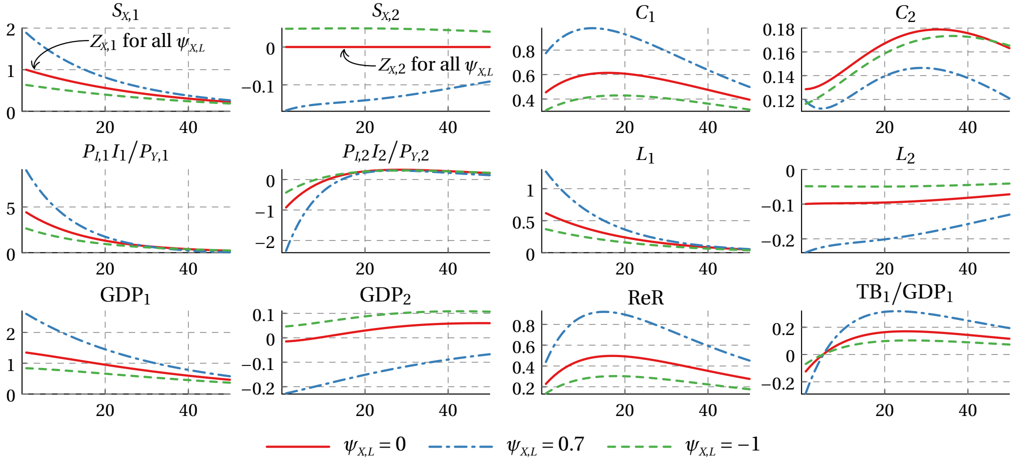
Role of Capital Externalities

Intuition for negative externality in capital

- Negative externality in capital makes the productivity faced by firms more transitory (with the same impact effect)
- Effects on GDP, labor, investment more transitory
- Due to consumption smoothing, as output is less persistent in future, consumption responds by less today and in the future
- This implies a higher response of investment today
- Also leads to a higher response of labor at home today because of a stronger substitution effect of wage increase compared to income effect
- This generates a higher response of output today
- Finally, endogenous positive correlation with foreign productivity also helps with co-movement

Role of Labor Externalities

Responses to a 1% home intermediate good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

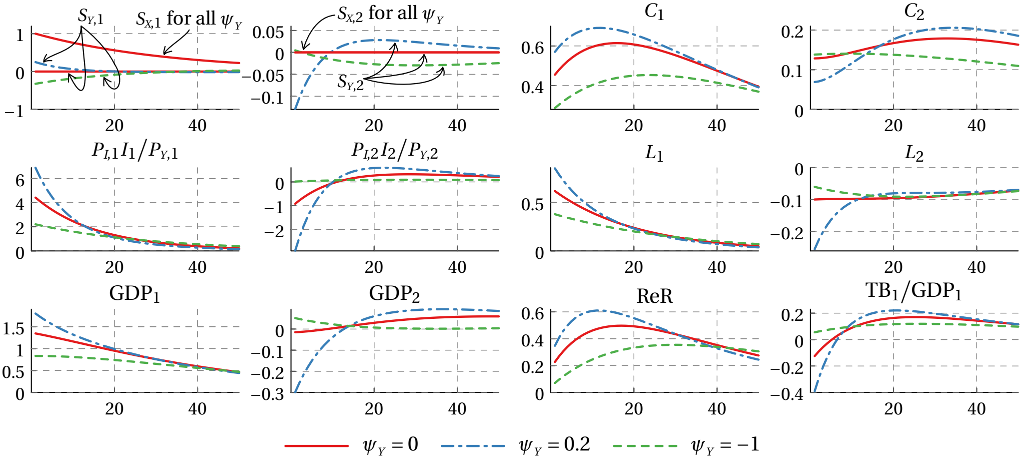
Role of Labor Externalities

Intuition for negative externality in labor

- Negative externality in labor makes the productivity faced by firms more transitory (but also with lower impact effect)
- Effects on GDP, labor, investment more transitory
- Due to consumption smoothing consumption responds by less today and in the future
- GDP, labor, and investment respond less today as well (current impact on productivity lower and so different from negative capital externality)
- As a result, net exports more procyclical and consumption correlation higher
- Endogenous positive correlation with foreign productivity also helps with international co-movement

Role of Final Good Externalities

Responses to a 1% home intermediate good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

Final Good Productivity Shock

- We introduce a final good productivity shock ($Z_{Y,nt}$)

$$Y_{nt} = Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y} \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- This shock is uncorrelated with the intermediate good productivity shock
- This additionally illustrates the changes in transmission mechanism from externalities
- Also, is useful quantitatively to match (gross) trade flows and net exports moments

Final Good Shock: Role of External Economies of Scale

Moment	Data	IRBC	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_{Y} 0.2	ψ_{Y} -1
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	-0.17	-0.21	$-0.08\uparrow$	-0.21	-0.18	-0.50	0.21
Corr (C_1, C_2)	0.36	-0.10	0.03	$-0.33\downarrow$	-0.06	-0.19	-0.34	0.19
Corr (I_1, I_2)	0.30	-0.62	-0.68	$-0.53\uparrow$	-0.57	-0.69	-0.83	-0.20
Corr (L_1, L_2)	0.42	-0.22	-0.42	$0.01\uparrow$	-0.28	-0.16	-0.59	0.24
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	-0.49	-0.69	-0.66	-0.73	-0.68	-0.71	-0.79	-0.56
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.74	0.77	$0.58\downarrow$	0.72	0.75	0.79	0.62

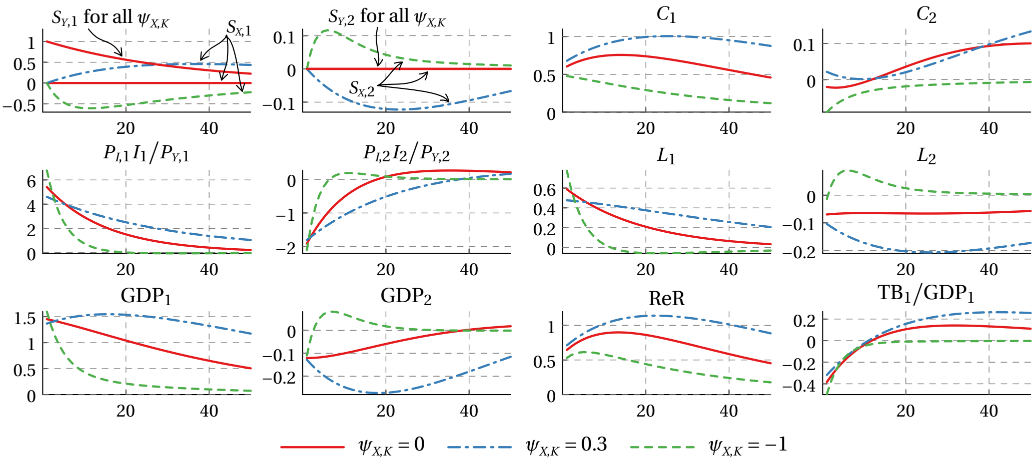
$$X_{nt} = \left[K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right] K_{X,nt}^{\alpha_X} L_{X,nt}^{1-\alpha_X}, \quad Y_{nt} = Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} X_{ni,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$\begin{bmatrix} \log(Z_{Y,1t}) \\ \log(Z_{Y,2t}) \end{bmatrix} = \begin{bmatrix} 0.97 & 0 \\ 0 & 0.97 \end{bmatrix} \times \begin{bmatrix} \log(Z_{Y,1,t-1}) \\ \log(Z_{Y,2,t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_{Y,1t} \\ \varepsilon_{Y,2t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{Y,1t} \\ \varepsilon_{Y,2t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.0073^2 & 0 \\ 0 & 0.0073^2 \end{bmatrix}\right)$$

Standard IRBC Calibration

Final Good Shock: Role of Capital Externalities

Responses to a 1% home final good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

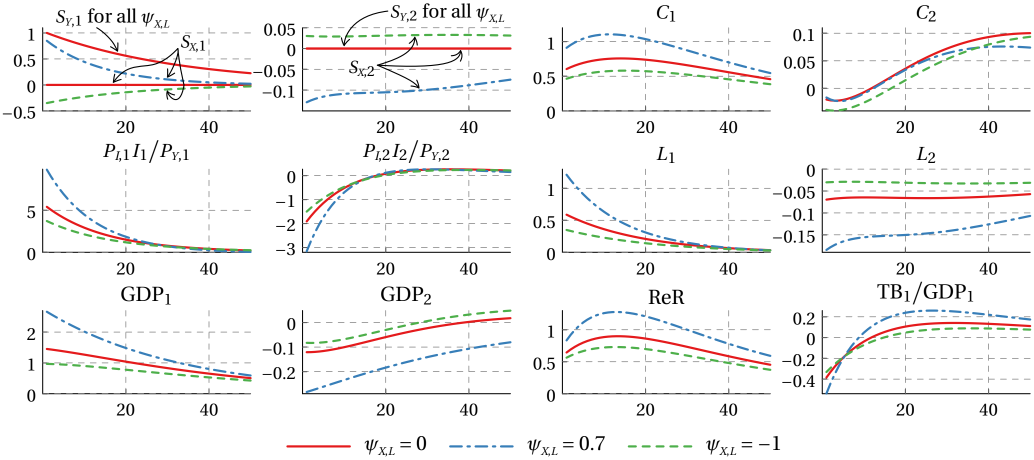
Final Good Shock: Role of Capital Externalities

Intuition for negative externality in capital

- First, with no externality, a positive shock leads to co-movement of home GDP, hours, investment, and consumption
- The key effect comes through direct impact on the real exchange rate
- As a result, there is a negative effect on foreign consumption
- Negative externality in capital leads to an endogenous negative effect on intermediate good productivity
- The overall transmission on home GDP, hours, and investment then looks similar to that of a shock to intermediate good productivity

Final Good Shock: Role of Labor Externalities

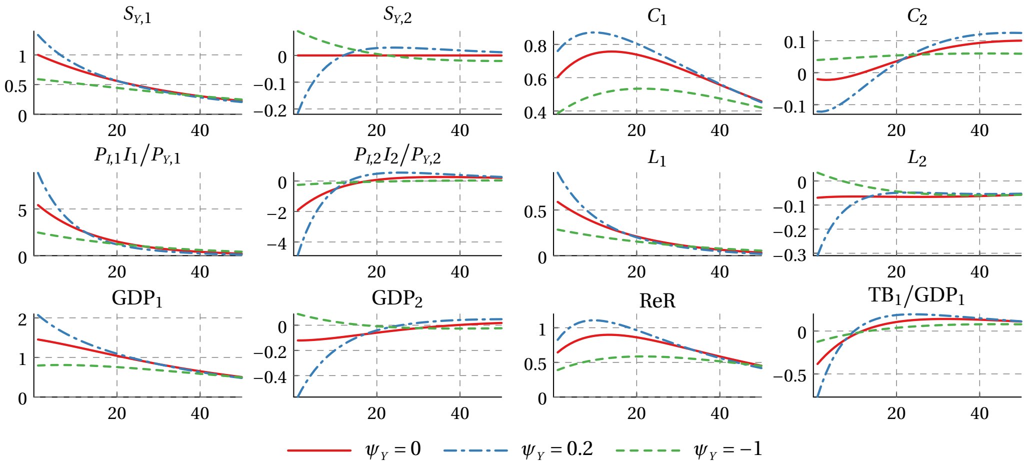
Responses to a 1% home final good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

Final Good Shock: Role of Final Good Externalities

Responses to a 1% home final good productivity shock



Notes: All horizontal axes are quarters after the shock. Vertical axes for the current account and trade balance measure the number of percentage points. All other vertical axes measure percent deviation from steady state.

Quantitative Exercise

- Consider a wider range of moments
  - Including domestic correlations and volatilities
- Use both shocks
- Estimate shock parameters and externalities to achieve best fit
  - Criterion is mean squared error
  - Other parameters same as in baseline IRBC calibration
- **Two main goals:**
  - Show that externalities need to be unrestricted to match data
  - Critical role played by negative capital externalities

Estimation Results

Moment	Data	Model	Moment	Data	Model
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	0.50	Corr (L_1, GDP_1)	0.87	1.00
Corr (C_1, C_2)	0.36	0.37	Corr $(P_{I,1}I_1/P_{Y,1}, \mathrm{GDP}_1)$	0.95	0.99
Corr $(P_{I,1}I_1/P_{Y,1}, P_{I,2}I_2/P_{Y,1})$	0.30	0.22	Std (GDP_1)	1.67	1.85
Corr (L_1, L_2)	0.42	0.50	Std (C_1)/Std (GDP_1)	0.81	0.15
Corr $(\mathrm{TB}_1/\mathrm{GDP}_1, \mathrm{GDP}_1)$	-0.49	-0.50	Std (L_1)/Std (GDP_1)	0.66	0.59
Corr $(\mathrm{Exp}_1, \mathrm{GDP}_1)$	0.32	0.45	Std $(P_{I,1}I_1/P_{Y,1})$/Std (GDP_1)	2.84	3.86
Corr $(\mathrm{Imp}_1, \mathrm{GDP}_1)$	0.81	0.99	Std (Exp_1)	3.94	1.92
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.15	Std (Imp_1)	5.42	1.97
Std $(\mathrm{TB}_1/\mathrm{GDP}_1)$	0.45	0.33	Std (ReR)/Std (GDP_1)	2.23	0.23
Corr (C_1, GDP_1)	0.86	0.96			

Parameter estimates:

$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	σ_X	σ_Y	ρ_X	ρ_Y
-2.70	0.91	-0.06	0.000	0.002	0.00	0.99

Non-targeted moments are in red Best Fit IRBC $\sigma = 2$

Conclusion

- Isomorphism between dynamic international trade and IRBC models allows an exploration of new mechanisms transparently
- Dynamic trade models do not generate significantly different results from the IRBC model because they imply relatively small, positive, and restricted externalities
- In order to improve fit with the data on international correlations, need negative external economies in capital
  - *Missing negative externality* puzzle

Future Work

- "*Monetary Policy in an International Business Cycle Model with Externalities*"
  - Isomorphism in an environment with sticky prices
  - Quantitative role of monetary policy shocks
- "*Growth, Externalities, and Technology Upgrading in Open Economies*"
  - Isomorphism between several popular classes of endogenous growth models
  - Role of externalities for balanced growth path, transition, and convergence

Appendix

Model: Households

- Very standard
- For complete financial markets:

$$\max_{\{C_{nt}, L_{nt}, I_{nt}, B_{n,t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_{nt}, L_{nt})$$

s.t.

$$P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + A_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt} + B_{nt},$$
$$A_{nt} = E_t \left[P_{B,t+1} B_{n,t+1}\right],$$
$$K_{n,t+1} = (1-\delta) K_{nt} + I_{nt},$$

where

- $B_{n,t+1}$ is nominal state-contingent period-$(t+1)$ bond, and $P_{B,t+1}$ is its price
- A_{nt} is net foreign asset position
- $P_{I,nt}$ is price of investment good
- Standard law of motion of capital

IRBC Model

Dynamic Version of Standard Krugman Model

Varieties and final aggregate

- Each country i produces a unique set of varieties Ω_{it}
  - Endogenously determined in every period t
  - M_{it} is the measure
- Each country buys all varieties from all other countries
  - Iceberg trade costs: $\tau_{ni,t}$
- Final aggregate is produced by combining varieties:

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[\left[\int_{\nu \in \boldsymbol{\Omega_{it}}} x_{ni,t}(\nu)^{\frac{\sigma^K - 1}{\sigma^K}} d\nu \right]^{\frac{\sigma^K}{\sigma^K - 1}} \right]^{\frac{\eta^K - 1}{\eta^K}} \right]^{\frac{\eta^K}{\eta^K - 1}}$$

  - $x_{ni,t}(\nu)$ is amount of variety $\nu \in \Omega_{it}$ that n buys from i
  - In corresponding IRBC: $\sigma = \eta^K$

Krugman Model

Dynamic Version of Standard Krugman Model

Production of varieties

- Production technology of variety $\nu \in \Omega_{nt}$:

$$x_{nt}(\nu) = Z_{x,nt} l_{nt}(\nu)$$

  - $l_{nt}(\nu)$ is the amount of labor in production of ν
  - $Z_{x,nt}$ is shock
- In order to enter the economy, producer of a variety in country n in period t needs to pay sunk cost equal to $W_{nt}/Z_{x,nt}$
  - W_{nt} is wage
  - $Z_{x,nt}$ is the same shock as in production technology
  - There is free entry: Value of variety = sunk cost
- Law of motion of varieties is

$$M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}$$

  - $M_{I,nt}$ is the number of producers of varieties that enter into the country n's economy in period t
  - δ is exogenous exit rate of varieties

Dynamic Version of Standard Melitz Model

Varieties

- Country i can produce any of the varieties from set Ω_{it}
  - Ω_{it} is endogenously determined
  - M_{it} is the measure
- All varieties from Ω_{it} can be internationally traded, but not all of them are available in a particular country n
  - $\Omega_{ni,t}$ is subset of country-i's varieties available in country n
  - $\Omega_{ni,t} \subseteq \Omega_{it}$
  - $\Omega_{ni,t}$ is endogenously determined
- Two types of trade costs:
  - Per-unit, iceberg: $\tau^{\text{M}}_{ni,t}$
  - Fixed: $\Phi_{ni,t}$

Dynamic Version of Generalized Melitz Model

Final aggregate

- Final aggregate is produced by combining varieties:

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[\left[\int_{\nu \in \boldsymbol{\Omega}_{ni,t}} x_{ni,t}(\nu)^{\frac{\sigma^{M}-1}{\sigma^{M}}} d\nu \right]^{\frac{\sigma^{M}}{\sigma^{M}-1}} \right]^{\frac{\eta^{M}-1}{\eta^{M}}} \right]^{\frac{\eta^{M}}{\eta^{M}-1}}$$

  - $x_{ni,t}(\nu)$ is amount of variety $\nu \in \Omega_{it}$ that n buys from i
  - In the corresponding IRBC:

$$\sigma - 1 = \frac{\theta^{M}}{\left(\frac{1}{\eta^{M}-1} - \frac{1}{\sigma^{M}-1} \right) \theta^{M} + 1}$$

Melitz Model

Dynamic Version of Generalized Melitz Model

Production of varieties

- Production technology of $\nu \in \Omega_{nt}$:

$$x_{nt}(\nu) = Z_{x,nt} \left[L^{\mathrm{M}}_{x,nt} \right]^{\phi_{x,L}} z_n(\nu)\, l_{nt}(\nu)$$

  - $l_{nt}(\nu)$ is the amount of labor in production of ν
  - $z_n(\nu)$ is efficiency of production of ν
  - $Z_{x,nt}$ is shock
  - $L^{\mathrm{M}}_{x,nt}$ is total amount of labor in production of varieties
    - Taken as given
    - Maps into quantity proportional to $L_{x,nt}$ in unified model
  - $\phi_{x,L}$ drives the external economies of scale

- Monopolistic competition in production of varieties

Dynamic Version of Generalized Melitz Model

Entry and exit of producers of varieties

- Sunk cost of entry equal to $\dfrac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$

- Law of motion of varieties is $M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}$

- Upon entry, producer of a new variety in country n gets an idiosyncratic efficiency draw, $z_n(\nu)$, from Pareto distribution with shape θ^{M} and minimal efficiency $z_{min,n}$:

$$Prob\left[z_n(\nu) \leq z\right] = 1 - \left(\frac{z_{min,n}}{z}\right)^{\theta^{M}}$$

Dynamic Version of Generalized Melitz Model

Fixed costs of serving markets

- To access country-n's market, country-i's producer of a variety has to pay fixed cost $\Phi_{ni,t}$ in terms of country-n's labor
- We posit that

$$\Phi_{ni,t} \equiv \left[M_{it}^{\frac{1}{\theta^{M}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}} F_{ni,t} \text{ with } \vartheta \equiv \frac{1}{\sigma^{M} - 1} - \frac{1}{\theta^{M}}$$

  - $L_{F,nt}$ is the total amount of country n's labor that is used to pay the fixed cost of serving its market
  - $F_{ni,t}$ is an exogenous part of the fixed cost
  - $M_{it}^{\frac{1}{\theta^{M}} - \phi_{F,M}}$ corrects the selection effect
    - Source of economies of scale in production of varieties
  - $L_{F,nt}^{\vartheta - \phi_{F,L}}$ corrects for the externality that arises due to interaction of love-of-variety and selection effects
    - Source of economies of scale in production of final aggregate

Dynamic Version of Generalized Melitz Model

Households

- For complete financial markets:

$$\max_{\{C_{nt}, L_{nt}, M_{I,nt}, B_{n,t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_{nt}, L_{nt})$$

s.t.

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} + A_{nt} = W_{nt} L_{nt} + D_{nt} M_{nt} + B_{nt},$$

$$A_{nt} = E_t \left[P_{B,t+1} B_{n,t+1} \right],$$

$$M_{n,t+1} = (1 - \delta) M_{nt} + M_{I,nt}$$

where

- $B_{n,t+1}$ is nominal state-contingent period-$(t+1)$ bond, and $P_{B,t+1}$ is its price
- A_{nt} is net foreign asset position
- V_{nt} is value of varieties
- D_{nt} is profits from producing varieties
- **Law of motion of varieties similar to that of capital**

Equilibrium System of Equations (1/2)

$$P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\},$$

$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}},$$

$$K_{n,t+1} = (1-\delta) K_{nt} + I_{nt},$$

$$X_{nt} = \Theta_{X,n} Z_{X,nt} K_{nt}^{\alpha_X + \psi_{X,K}} L_{X,nt}^{1-\alpha_X + \psi_{X,L}},$$

$$Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

$$I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I},$$

$$W_{nt} L_{X,nt} + W_{nt} L_{I,nt} = W_{nt} L_{nt} + a\mathrm{TB}_{nt},$$

$$C_{nt} + Y_{I,nt} = Y_{nt},$$

Equilibrium System of Equations (2/2)

$$\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it},$$

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{1-\sigma}},$$

$$K_{nt} = \alpha_X \frac{P_{X,nt} X_{nt}}{R_{nt}},$$

$$L_{X,nt} = (1-\alpha_X) \frac{P_{X,nt} X_{nt}}{W_{nt}},$$

$$L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}},$$

$$Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.$$

Standard IRBC, Spillovers and Correlations: Moments

		IRBC		
Moment	Data	Complete	Bond	Fin. aut.
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	0.14	0.17	0.29
Corr (C_1, C_2)	0.36	0.79	0.65	0.68
Corr (I_1, I_2)	0.30	-0.48	-0.45	-0.02
Corr (L_1, L_2)	0.42	-0.51	-0.38	-0.23
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	-0.49	-0.49	-0.55	
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.53	0.46	0.60

IRBC Model Moments

Changing σ in Standard IRBC: Moments

No spillovers, no correlations, complete markets

Moment	Data	$\sigma = 0.9$	$\sigma = 2$	$\sigma = 6$
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	0.25	−0.03	−0.27
Corr (C_1, C_2)	0.36	0.21	0.47	0.69
Corr (I_1, I_2)	0.30	−0.06	−0.39	−0.77
Corr (L_1, L_2)	0.42	0.28	−0.30	−0.61
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.60	−0.49	−0.48
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.58	0.61	0.05

IRBC Model Moments

IRBC and Melitz Model Moments

Moment	Data	IRBC	IRBC Inv. labor	Mel	Mel Inv. final
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	−0.03	0.07	−0.09	−0.11
Corr (C_1, C_2)	0.36	0.47	0.58	0.45	0.37
Corr (I_1, I_2)	0.30	−0.39	0.12	−0.26	−0.44
Corr (L_1, L_2)	0.42	−0.30	−0.84	−0.46	−0.43
Corr $\left(\frac{\mathrm{TB}_1}{\mathrm{GDP}_1}, \mathrm{GDP}_1\right)$	−0.49	−0.49	0.68	0.61	−0.20
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.61	0.68	0.67	0.69

Melitz Model Moments

Best Fit: $\sigma = 2$, IRBC ($\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$, $\sigma_Y = 0$)

Moment	Data	Model	Moment	Data	Model
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	0.02	Corr (L_1, GDP_1)	0.87	1.00
Corr (C_1, C_2)	0.36	0.10	Corr $(P_{I,1}I_1/P_{Y,1}, \mathrm{GDP}_1)$	0.95	0.98
Corr $(P_{I,1}I_1/P_{Y,1}, P_{I,2}I_2/P_{Y,1})$	0.30	−0.32	Std (GDP_1)	1.67	1.67
Corr (L_1, L_2)	0.42	0.01	Std (C_1)/Std (GDP_1)	0.81	0.17
Corr $(\mathrm{TB}_1/\mathrm{GDP}_1, \mathrm{GDP}_1)$	−0.49	−0.67	Std (L_1)/Std (GDP_1)	0.66	0.58
Corr $(\mathrm{Exp}_1, \mathrm{GDP}_1)$	0.32	−0.05	Std $(P_{I,1}I_1/P_{Y,1})$/Std (GDP_1)	2.84	4.20
Corr $(\mathrm{Imp}_1, \mathrm{GDP}_1)$	0.81	0.98	Std (Exp_1)	3.94	1.89
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.35	Std (Imp_1)	5.42	1.93
Std $(\mathrm{TB}_1/\mathrm{GDP}_1)$	0.45	0.45	Std (ReR)/Std (GDP_1)	2.23	0.20
Corr (C_1, GDP_1)	0.86	0.94			

Parameter estimates:

σ_X	ρ_X
0.011	0.34

Non-targeted moments are in red

Best Fit $\sigma = 2$ | Fixed Persistence

Best Fit: $\sigma = 2$, $\psi_{X,L} = \psi_Y = 0$, $\rho_X = 0.97$, $\sigma_Y = 0$

Moment	Data	Model	Moment	Data	Model
Corr $(\mathrm{GDP}_1, \mathrm{GDP}_2)$	0.58	0.10	Corr (L_1, GDP_1)	0.87	1.00
Corr (C_1, C_2)	0.36	0.25	Corr $(P_{I,1}I_1/P_{Y,1}, \mathrm{GDP}_1)$	0.95	0.98
Corr $(P_{I,1}I_1/P_{Y,1}, P_{I,2}I_2/P_{Y,1})$	0.30	−0.25	Std (GDP_1)	1.67	1.71
Corr (L_1, L_2)	0.42	0.08	Std (C_1)/Std (GDP_1)	0.81	0.19
Corr $(\mathrm{TB}_1/\mathrm{GDP}_1, \mathrm{GDP}_1)$	−0.49	−0.63	Std (L_1)/Std (GDP_1)	0.66	0.57
Corr $(\mathrm{Exp}_1, \mathrm{GDP}_1)$	0.32	0.05	Std $(P_{I,1}I_1/P_{Y,1})$/Std (GDP_1)	2.84	4.08
Corr $(\mathrm{Imp}_1, \mathrm{GDP}_1)$	0.81	0.97	Std (Exp_1)	3.94	1.88
Corr $(\mathrm{ReR}, \mathrm{GDP}_1)$	0.13	0.36	Std (Imp_1)	5.42	1.92
Std $(\mathrm{TB}_1/\mathrm{GDP}_1)$	0.45	0.42	Std (ReR)/Std (GDP_1)	2.23	0.24
Corr (C_1, GDP_1)	0.86	0.92			

Parameter estimates:

$\psi_{X,K}$	σ_X
−3.09	0.011

Non-targeted moments are in red. Best Fit IRBC