Approximate Expected Utility Rationalization

Federico Echenique
Taisuke Imai
Kota Saito

Motivation

- Revealed preference theory asks

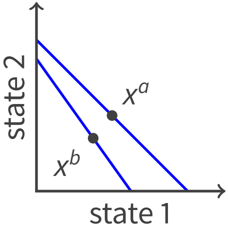
  When are agent's choices consistent with utility maximization?

  $\rightsquigarrow$ about general utility maximization

- Recent theory is about specific functional forms
- This paper
  - expected utility
  - practical tool for experiments on choice under risk and uncertainty

Motivation

$$\max \sum_{s \in S} \mu_s u(x_s)$$
$$\text{s.t.} \sum_{s \in S} p_s x_s \leqslant I$$



- When are choices from budget sets consistent with EU?
- Can we find u (and μ) such that for all $k \in \{a, b\}$

$$(x_1^k, x_2^k) \in \underset{(x_1, x_2)}{\text{argmax}} \; \mu_1 u(x_1) + \mu_2 u(x_2)$$
$$\text{s.t.} \quad p_1^k x_1 + p_2^k x_2 \leqslant p_1^k x_1^k + p_2^k x_2^k$$

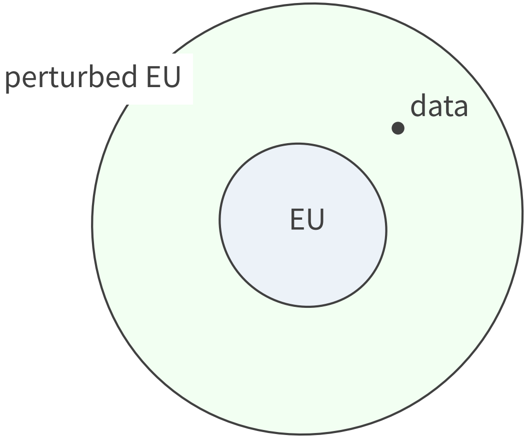
Motivation

- Characterize EU in market setting: answers known
  - Green and Srivastava (1986): objective EU (OEU)
  - Kubler, Selden, and Wei (2014): OEU
  - Echenique and Saito (2015): subjective EU (SEU), OEU
  - Polisson, Quah, and Renou (2020): OEU, SEU, $\cdots$
- We can test EU using these characterizations
- What if data is not consistent with EU?

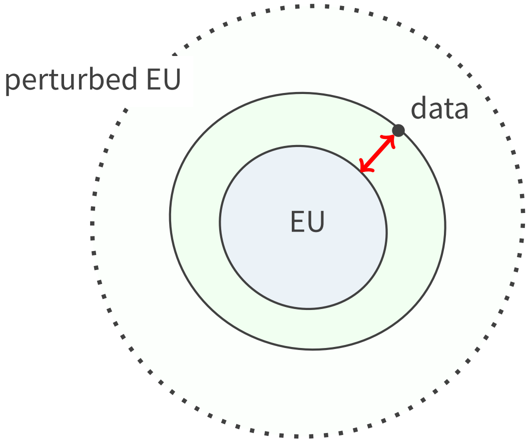
Preview

- Introduce and characterize "perturbed" EU
  - "concave" OEU / SEU (assume risk aversion)
  - revealed preference approach
- Develop a measure of consistency with EU $\rightsquigarrow$ practical tool
  - "how far" is a given choice data from EU?

Preview



Preview



Preview

- Apply the method to data from three large experiments involving risk
  - heterogeneity
  - compare with other measures of rationality
  - demographic characteristics

younger subjects	$\rightsquigarrow$	closer to OEU
higher education	$\rightsquigarrow$	closer to OEU
higher cognitive ability	$\rightsquigarrow$	closer to OEU

Warmup

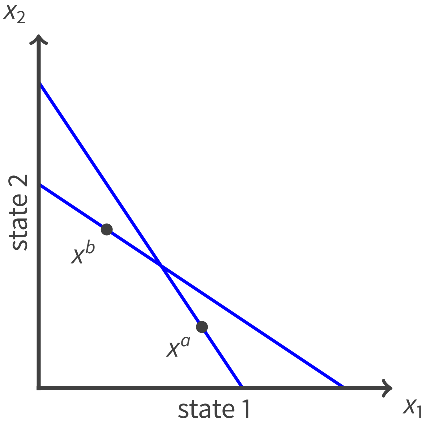
2×2 example

- 2×2 case
  - 2 states
  - 2 observations
- What is the meaning of

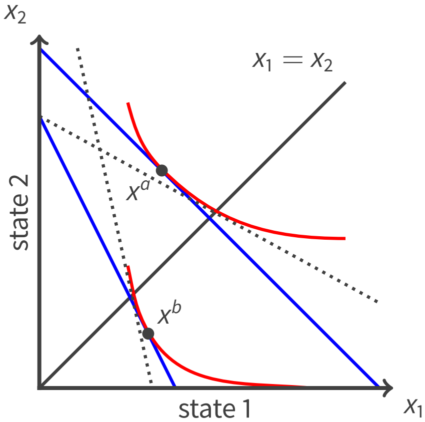
$$
\begin{aligned}
\max \quad & \mu_1 u(x_1) + \mu_2 u(x_2) \\
\text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leqslant I
\end{aligned}
$$

2×2 example

- Violation of WARP



2×2 example



- OEU agent solves

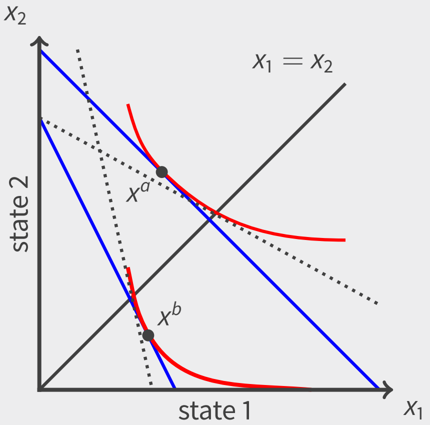
$$\begin{aligned} \max \quad & \mu_1 u(x_1) + \mu_2 u(x_2) \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leqslant I \end{aligned}$$

- $\text{MRS}|_{x_1 = x_2} = \dfrac{\mu_1 u'(x_1)}{\mu_2 u'(x_2)} = \dfrac{\mu_1}{\mu_2}$

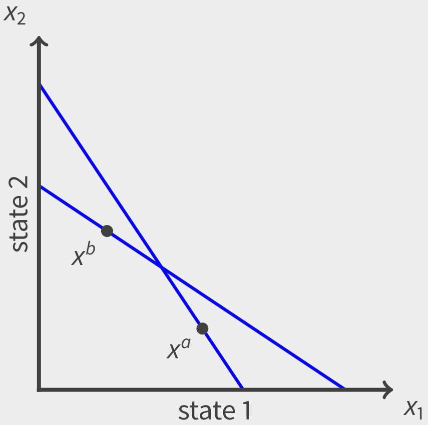
  ... but ········· have different slopes

2×2 example

Not



Not



- First-order conditions

$$\mu_s u'(x_s) = \lambda p_s \quad \text{for all} \quad s \quad \Longrightarrow \quad \frac{\mu_1 u'(x_1)}{\mu_2 u'(x_2)} = \frac{p_1}{p_2}$$

- Downward-sloping demand property
  - state-to-state relatively higher prices relate to smaller quantities

$$p_1 \nearrow \quad \Longrightarrow \quad u'(x_1) \nearrow \quad \Longrightarrow \quad x_1 \searrow$$

Model

Setup for OEU

- State: $S = \{1, \dots, n\}$
- Objective probability: $\mu^* = (\mu_1^*, \dots, \mu_n^*) \in \Delta_{++}(S)$
- State-contingent monetary payoffs: $\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbf{R}_+^{|S|}$
  - $\boldsymbol{x}$ pays x_s in state s
- Prices: $\boldsymbol{p} = (p_1, \dots, p_n) \in \mathbf{R}_{++}^{|S|}$
- Observation: $(\boldsymbol{x}^k, \boldsymbol{p}^k)$, $k \in \mathcal{K} = \{1, \dots, K\}$
  - $x_s^k \in \mathbf{R}_+$: payoff in state s in observation k
  - $p_s^k \in \mathbf{R}_{++}$: price in state s in observation k

Definition

A *dataset* is a collection $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$, where $\boldsymbol{x}^k \in \mathbf{R}_+^{|S|}$ and $\boldsymbol{p}^k \in \mathbf{R}_{++}^{|S|}$ for all $k \in \mathcal{K}$.

OEU rationalization

Definition

A dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is *Objective Expected Utility (OEU) rational* if there exists a concave and strictly increasing $u : \mathbf{R}_+ \to \mathbf{R}$ such that for all $k \in \mathcal{K}$ and all $\boldsymbol{y} \in \mathbf{R}_+^{|S|}$,

$$\sum_{s \in S} p_s^k y_s \leqslant \sum_{s \in S} p_s^k x_s^k \implies \sum_{s \in S} \mu_s^* u(y_s) \leqslant \sum_{s \in S} \mu_s^* u(x_s^k)$$

- Interpretation: agent solves $\max \sum \mu_s^* u(y_s)$ subject to $\sum_s p_s^k y_s \leqslant I$ where $I = \sum_s p_s^k x_s^k$
- Notation: $B(\boldsymbol{p}, I) = \{\boldsymbol{y} : \boldsymbol{p} \cdot \boldsymbol{y} \leqslant I\}$

e-Perturbed OEU

- Suppose a dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is not OEU rational

$$\begin{aligned} \max_{\boldsymbol{x}^k} \quad & \sum_{s \in S} \mu_s^* u(x_s^k) \\ \text{s.t.} \quad & \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \end{aligned}, \quad k \in \mathcal{K}$$

e-Perturbed OEU

- Suppose a dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is not OEU rational

$$\left.\begin{aligned} \max_{\boldsymbol{x}^k} \quad & \sum_{s \in S} \mu_s^* u(x_s^k) \\ \text{s.t.} \quad & \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \end{aligned}\right. , \quad k \in \mathcal{K}$$

1. Belief perturbation: $\mu^* \rightsquigarrow \mu^k$

- Suppose a dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is not OEU rational

$$\begin{aligned} \max_{\boldsymbol{x}^k} \quad & \sum_{s \in S} \mu_s^* u(x_s^k) \\ \text{s.t.} \quad & \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \end{aligned}, \quad k \in \mathcal{K}$$

1. Belief perturbation: $\mu^* \rightsquigarrow \mu^k$
2. Utility perturbation: $u(x_s^k) \rightsquigarrow u(x_s^k)\varepsilon_s^k$

e-Perturbed OEU

- Suppose a dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is not OEU rational

$$\left.\begin{aligned} \max_{\boldsymbol{x}^k} \quad & \sum_{s \in S} \mu_s^* u(x_s^k) \\ \text{s.t.} \quad & \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \end{aligned}\right. , \quad k \in \mathcal{K}$$

1. Belief perturbation: $\mu^* \rightsquigarrow \mu^k$
2. Utility perturbation: $u(x_s^k) \rightsquigarrow u(x_s^k)\varepsilon_s^k$
3. Price perturbation: $p_s^k \rightsquigarrow p_s^k \varepsilon_s^k$

e-Perturbed OEU

- Fix a positive number e
- e-belief-perturbed OEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s^k u(x_s^k)$$
$$\text{s.t.} \quad \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k)$$

  - for each $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\mu_s^k / \mu_t^k}{\mu_s^* / \mu_t^*} \leqslant 1 + e$$

e-Perturbed OEU

- Fix a positive number e
- e-price-perturbed OEU

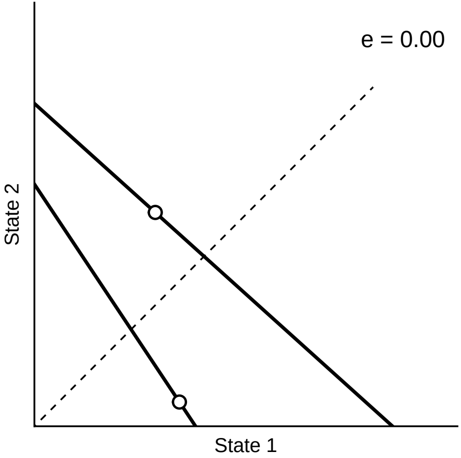
$$\max_{\boldsymbol{x}^k} \sum_{s\in S} \mu_s^* u(x_s^k)$$

$$\text{s.t.} \quad \boldsymbol{x}^k \in B(\tilde{\boldsymbol{p}}^k, \tilde{\boldsymbol{p}}^k \cdot \boldsymbol{x}^k)$$

$$\tilde{p}_s^k = p_s^k \varepsilon_s^k$$

  - for each $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1+e$$



- Fix a positive number e
- e-price-perturbed OEU

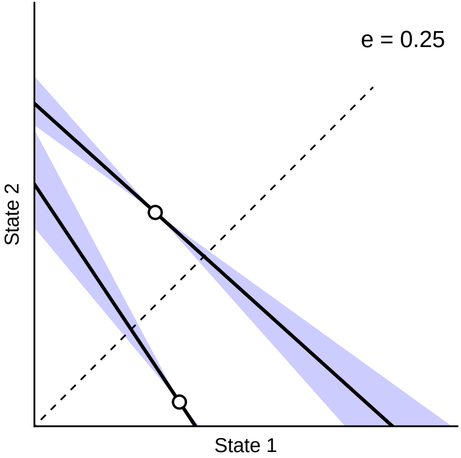
$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s^* u(x_s^k)$$

$$\text{s.t. } \boldsymbol{x}^k \in B(\tilde{\boldsymbol{p}}^k, \tilde{\boldsymbol{p}}^k \cdot \boldsymbol{x}^k)$$

$$\tilde{p}_s^k = p_s^k \varepsilon_s^k$$

  - for each $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1+e$$



- Fix a positive number e
- e-price-perturbed OEU

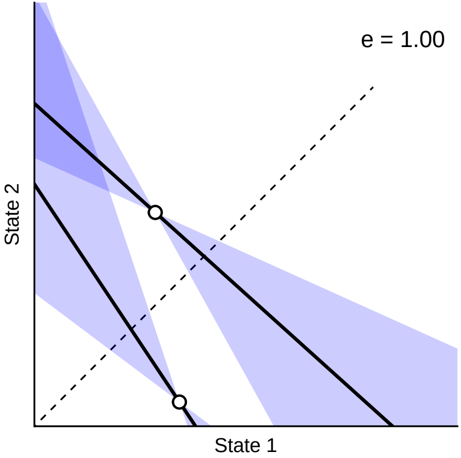
$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s^* u(x_s^k)$$

$$\text{s.t.} \quad \boldsymbol{x}^k \in B(\tilde{\boldsymbol{p}}^k, \tilde{\boldsymbol{p}}^k \cdot \boldsymbol{x}^k)$$

$$\tilde{p}_s^k = p_s^k \varepsilon_s^k$$

  - for each $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1 + e$$



e-Perturbed OEU

- Fix a positive number e
- e-price-perturbed OEU

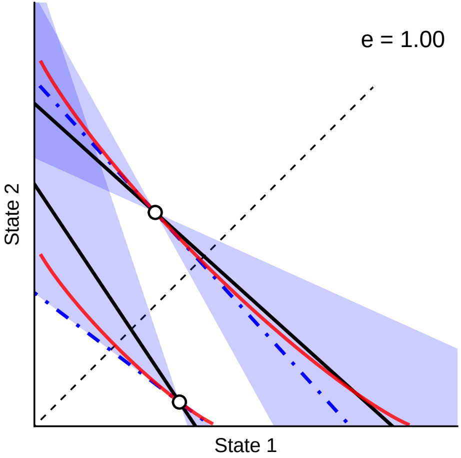
$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s^* u(x_s^k)$$

$$\text{s.t.} \quad \boldsymbol{x}^k \in B(\tilde{\boldsymbol{p}}^k, \tilde{\boldsymbol{p}}^k \cdot \boldsymbol{x}^k)$$

$$\tilde{p}_s^k = p_s^k \varepsilon_s^k$$

  - for each $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1+e$$



Equivalence

Theorem

Let $e \in \mathbf{R}_+$, and D be a dataset. The following statements are equivalent:

- D is e-belief-perturbed OEU rational,
- D is e-utility-perturbed OEU rational,
- D is e-price-perturbed OEU rational.

- Intuition: FOC

$$\mu_s^* u'(x_s^k) = \lambda^k p_s^k$$

- In the paper:
  - axiomatic characterization of e-perturbed OEU
  - equivalence and characterization of e-perturbed SEU

A measure of degree of consistency with EU

- Bounds on belief distortions in e-belief-perturbed OEU

$$\frac{1}{1+e} \leqslant \frac{\mu_s^k/\mu_t^k}{\mu_s^*/\mu_t^*} \leqslant 1+e$$

$e = 0$ $\mu_s^k = \mu_s^*$ for all $s \in S, k \in \mathcal{K}$ $\rightsquigarrow$ exact OEU rational

$e = \infty$ any data is perturbed OEU rationalizable

- What is the smallest e that makes non-OEU dataset e-perturbed OEU rational?

A measure of degree of consistency with EU

Definition

Minimal e, denoted e_*, is the smallest $e' \geqslant 0$ for which the data is e'-perturbed OEU rational.

Related measure of rationality

- Critical Cost Efficiency Index (CCEI) Afriat (1972)
  - how much budget sets need to be shifted to remove any violations of GARP
  - about general utility maximization
- EU-CCEI Polisson, Quah, and Renou (2020)
  - a version of CCEI for EU using the GRID method
  - no need to impose risk aversion
- Note:

e_* rotate budget lines to remove EU-violating obs

CCEI shift budget lines to remove GARP-violating obs

Application

Data

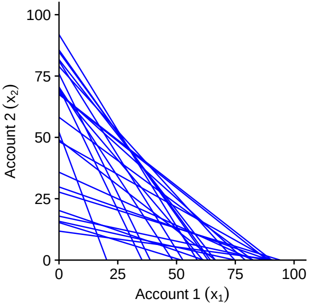
- Datasets:

  CKMS Choi, Kariv, Müller, and Silverman (2014) *AER*

  CMW Carvalho, Meier, and Wang (2016) *AER*

  CS Carvalho and Silverman (2019) NBER WP No. 26036

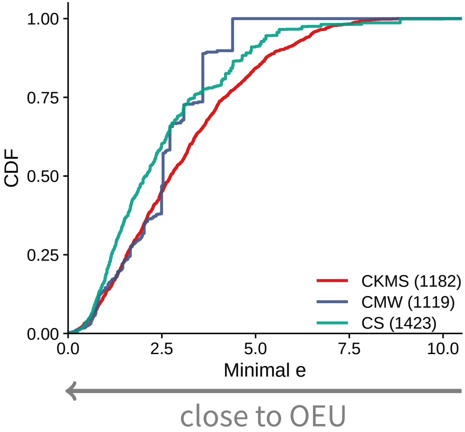
- Structure: choose state-contingent payoffs from 25 linear budgets, given objective probability (50-50)



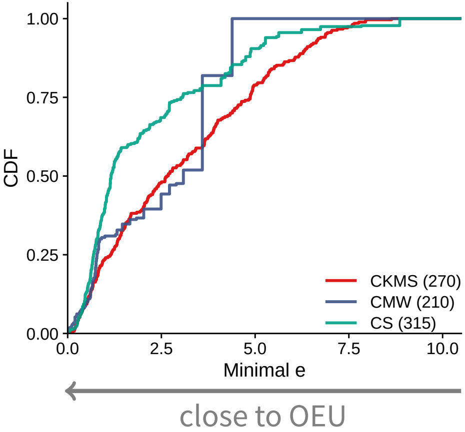
	# subjects	
	total	exact OEU
CKMS	1,182	0
CMW	1,119	3
CS	1,423	2

Distribution of e_*

- All subjects

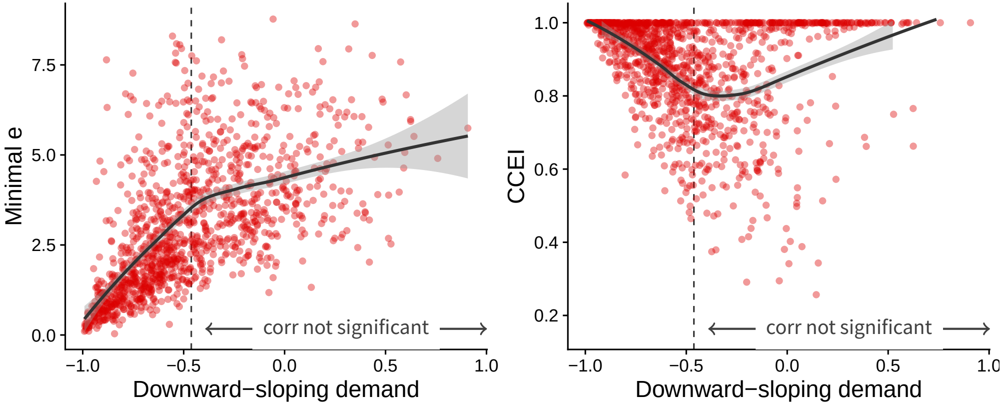


- Subjects with CCEI $= 1$



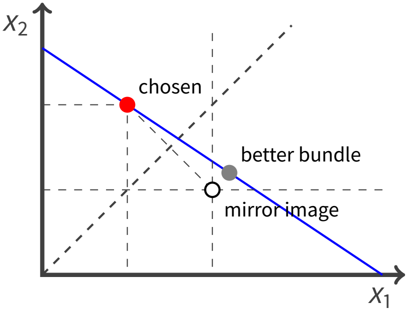
Downward-sloping demand

- Response to price changes: corr $\left(\log(p_2/p_1), \log(x_2/x_1)\right)$
- e_* measures deviation from downward-sloping demand

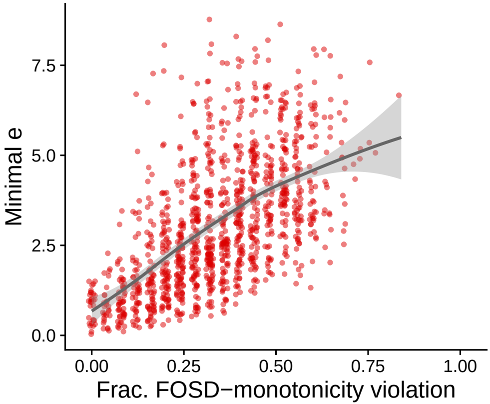


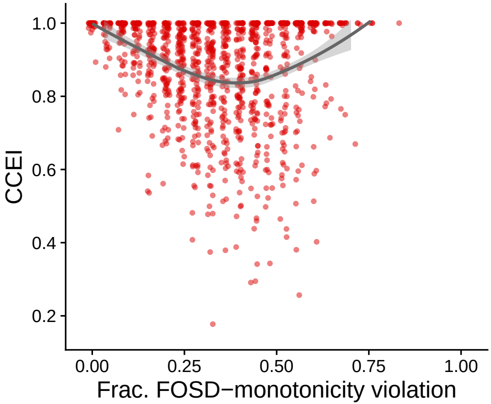
First-order stochastic dominance

- In 50-50 environment, choosing (x_1, x_2) at prices (p_1, p_2) violates monotonicity w.r.t. FOSD if
  - $p_1 > p_2$ and $x_1 > x_2$, or
  - $p_1 < p_2$ and $x_1 < x_2$
- Single-budget example: Choice ● violates FOSD-monotonicity (● $\sim$ ○ $\prec$ ●)



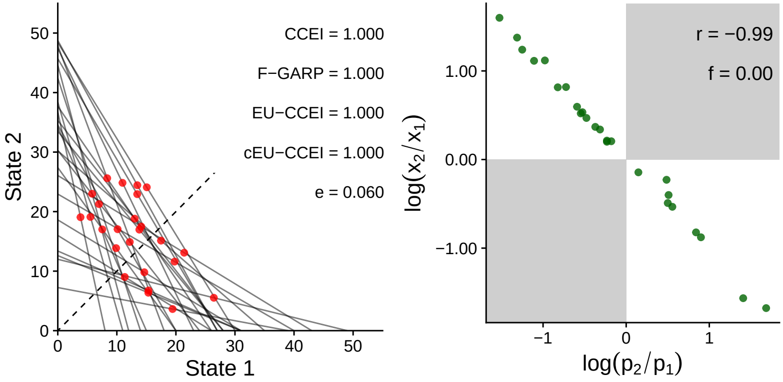
First-order stochastic dominance





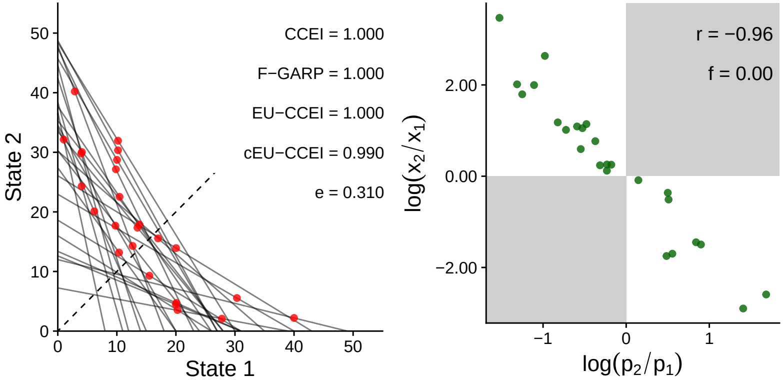
Sample choice pattern

- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation



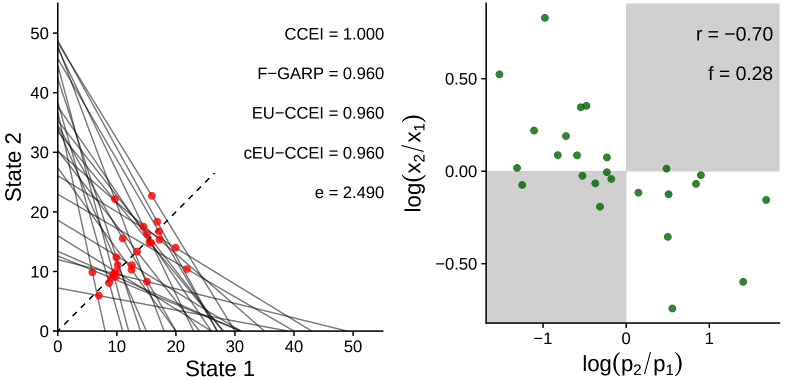
Sample choice pattern

- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation



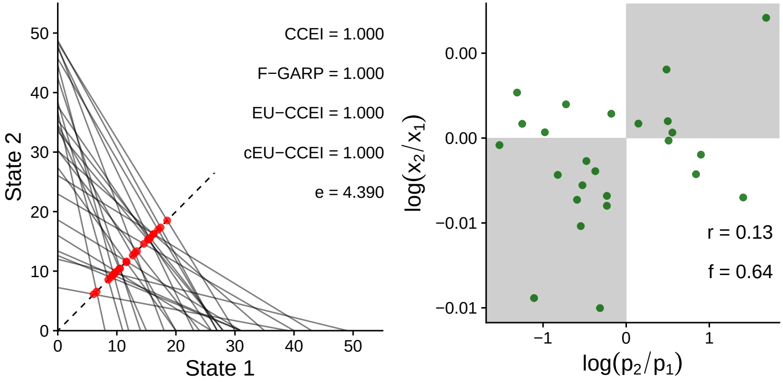
Sample choice pattern

- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation



Sample choice pattern

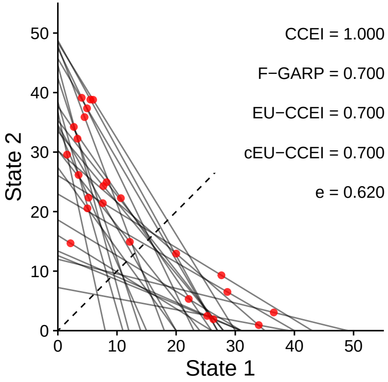
- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation

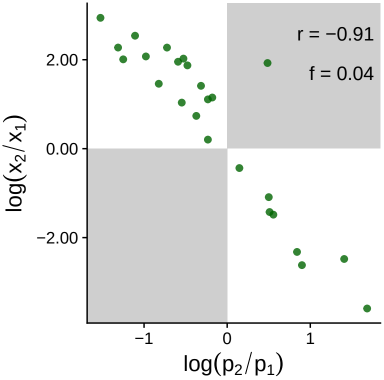


Note: Choices are almost, but not exactly, on the 45-degree line

Sample choice pattern

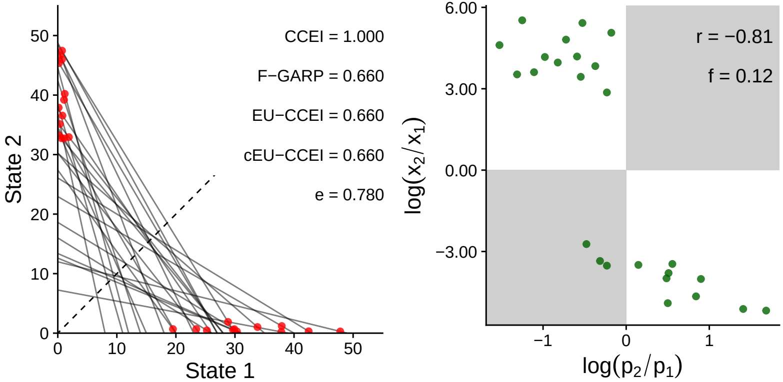
- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation





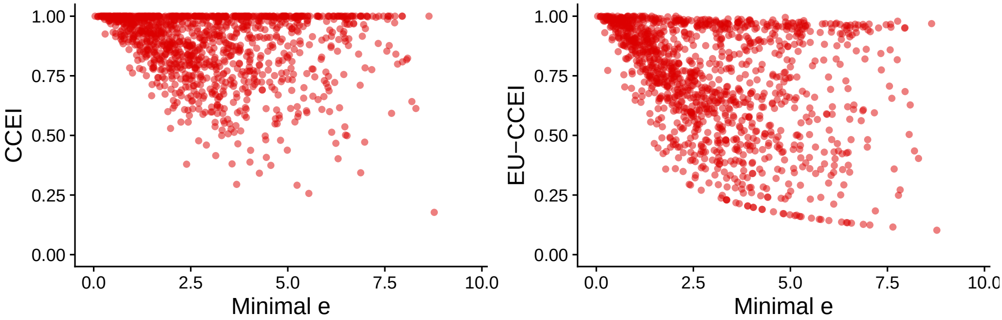
Sample choice pattern

- Choices, e_*, and other measures (subjects with CCEI $= 1$)
  - F-GARP, EU-CCEI, cEU-CCEI: GRID method Polisson, Quah, and Renou (2020)
  - r: downward-sloping demand, f: FOSD-mon violation

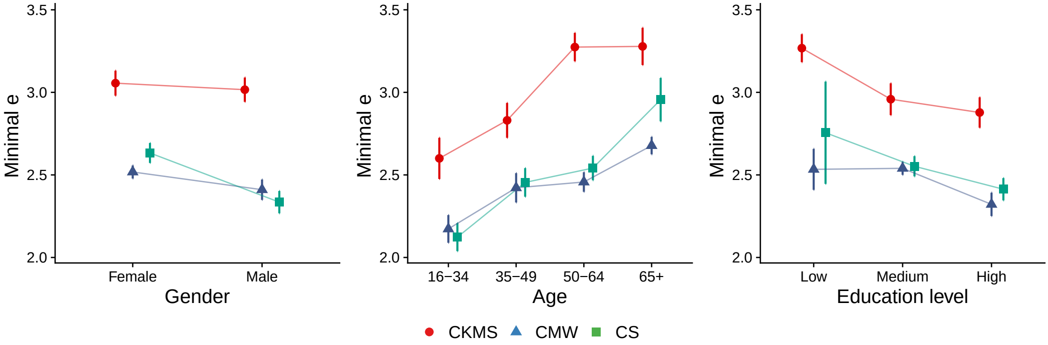


Correlation between two measures

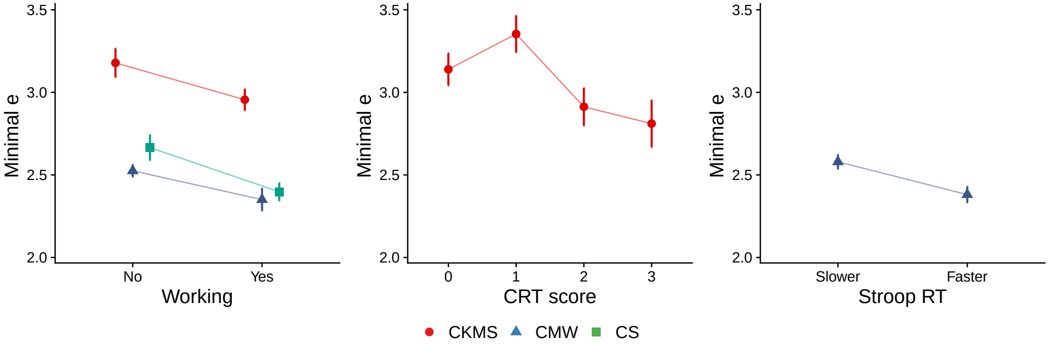
- Compare e_* and
    - CCEI: general utility maximization
    - EU-CCEI: EU maximization without imposing risk aversion
- Measures are correlated but provide different conclusions for many subjects



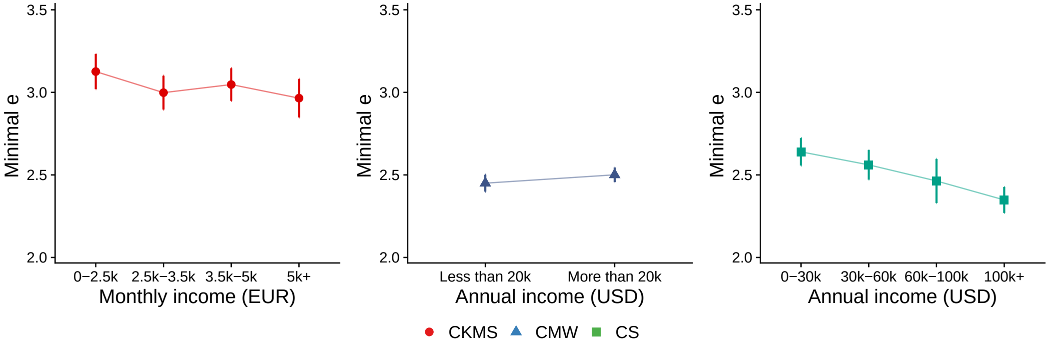
Demographic characteristics



Demographic characteristics



Demographic characteristics



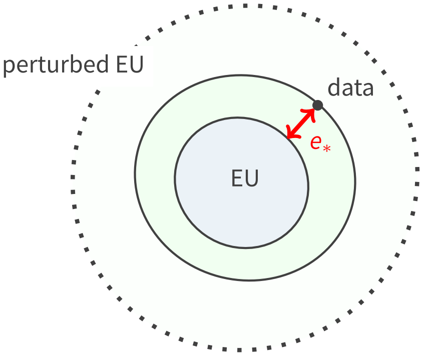
Additional results

- Understanding the size of e_*
  - comparison with random choice benchmark
  - statistical hypothesis testing
- Sensitivity
  - dropping 1 or 2 “critical” mistakes does not change results dramatically

Summary

What we did

- Introduce and characterize "*e*-perturbed" Objective EU and Subjective EU
- Develop a measure of consistency with EU, minimal *e*
- Develop a procedure for statistical hypothesis testing
- Apply the method using data from three large survey experiments



Additional materials

Contents

e-Perturbed OEU rationalization

Definition

Given $e \in \mathbf{R}_+$, a dataset $(x^k, p^k)_{k=1}^K$ is *e-belief-perturbed Objective Expected Utility rational* if there exist a concave and strictly increasing $u : \mathbf{R}_+ \to \mathbf{R}$ and $\mu^k \in \Delta_{++}(S)$ for each $k \in \mathcal{K}$, such that for all $k \in \mathcal{K}$ and all $\boldsymbol{y} \in \mathbf{R}_+^{|S|}$,

$$\sum_{s \in S} p_s^k y_s \leqslant \sum_{s \in S} p_s^k x_s^k \implies \sum_{s \in S} \mu_s^k u(y_s) \leqslant \sum_{s \in S} \mu_s^k u(x_s^k)$$

and for each $k \in \mathcal{K}$ and $s, t \in S$,

$$\frac{1}{1+e} \leqslant \frac{\mu_s^k / \mu_t^k}{\mu_s^* / \mu_t^*} \leqslant 1 + e$$

e-Perturbed OEU rationalization

Definition

Given $e \in \mathbf{R}_+$, a dataset $(x^k, p^k)_{k=1}^K$ is *e-price-perturbed Objective Expected Utility rational* if there exist a concave and strictly increasing $u : \mathbf{R}_+ \to \mathbf{R}$ and $\varepsilon^k \in \mathbf{R}_+^{|S|}$ for each $k \in \mathcal{K}$ such that for all $k \in \mathcal{K}$ and all $\boldsymbol{y} \in \mathbf{R}_+^{|S|}$,

$$\sum_{s \in S} \left(p_s^k \varepsilon_s^k\right) y_s \leqslant \sum_{s \in S} \left(p_s^k \varepsilon_s^k\right) x_s^k \implies \sum_{s \in S} \mu_s^* u(y_s) \leqslant \sum_{s \in S} \mu_s^* u(x_s^k)$$

and for each $k \in \mathcal{K}$ and $s, t \in S$,

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1 + e$$

e-Perturbed OEU rationalization

Definition

Given $e \in \mathbf{R}_+$, a dataset $(x^k, p^k)_{k=1}^K$ is *e-utility-perturbed Objective Expected Utility rational* if there exist a concave and strictly increasing $u : \mathbf{R}_+ \to \mathbf{R}$ and $\varepsilon^k \in \mathbf{R}_+^{|S|}$ for each $k \in \mathcal{K}$ such that for all $k \in \mathcal{K}$ and all $\boldsymbol{y} \in \mathbf{R}_+^{|S|}$,

$$\sum_{s \in S} p_s^k y_s \leqslant \sum_{s \in S} p_s^k x_s^k \implies \sum_{s \in S} \mu_s^* \left(u(y_s)\varepsilon_s^k\right) \leqslant \sum_{s \in S} \mu_s^* \left(u(x_s^k)\varepsilon_s^k\right)$$

and for each $k \in \mathcal{K}$ and $s, t \in S$,

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1 + e$$

Characterization

Definition

For any dataset $(p^k, x^k)_{k=1}^K$, the *risk neutral price* $\rho_s^k \in \mathbf{R}_{++}^{|S|}$ in choice problem k at state s is defined by $\rho_s^k = p_s^k / \mu_s^*$.

Definition

A sequence of pairs $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^m$ is called a *test sequence* if

1. $x_{s_i}^{k_i} > x_{s_i'}^{k_i'}$ for all $i = 1, \ldots, m$;
2. each k appears as k_i (on the left of the pair) the same number of times it appears as k_i' (on the right).

Characterization

Strong Axiom for Revealed OEU Echenique and Saito (2015)

For any test sequence of pairs $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^{m}$, we have

$$\prod_{i=1}^{m} \frac{\rho_{s_i}^{k_i}}{\rho_{s_i'}^{k_i'}} \leqslant 1$$

- Necessity: FOC

$$\lambda^k p_s^k = \mu_s^* u'(x_s^k) \iff \rho_s^k = u'(x_s^k)/\lambda^k$$

then concavity of u implies

$$\prod_{i=1}^{m} \frac{\rho_{s_i}^{k_i}}{\rho_{s_i'}^{k_i'}} = \prod_{i=1}^{m} \frac{\lambda^{k_i'}}{\lambda^{k_i}} \prod_{i=1}^{m} \frac{u'(x_{s_i}^{k_i})}{u'(x_{s_i'}^{k_i'})} \leqslant 1$$

Characterization

- SAROEU is not necessary for e-perturbed OEU rationality
- DM has belief μ^k for each problem k
- FOC

$$\lambda^k p_s^k = \mu_s^k u'(x_s^k) \quad \Longleftrightarrow \quad \rho_s^k = \frac{\mu_s^k}{\mu_s^*} \frac{u'(x_s^k)}{\lambda^k}$$

- Suppose $x_s^k > x_t^k$

$$\frac{\rho_s^k}{\rho_t^k} = \left(\frac{\mu_s^k}{\mu_s^*} \frac{u'(x_s^k)}{\lambda^k} \right) \Bigg/ \left(\frac{\mu_t^k}{\mu_t^*} \frac{u'(x_t^k)}{\lambda^k} \right) = \underbrace{\frac{u'(x_s^k)}{u'(x_t^k)}}_{\leqslant 1} \frac{\mu_s^k / \mu_s^*}{\mu_t^k / \mu_t^*} \leqslant 1 + e$$

Characterization

- Sequence of pairs $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^m$
- If x_s^k appears as $x_{s_i}^{k_i}$ for some i and as $x_{s_j'}^{k_j'}$ for some j, μ_s^k is cancelled out

Definition

Consider any sequence of pairs $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^m \equiv \sigma$. For any $k \in \mathcal{K}$ and $s \in S$,

$$d(\sigma, k, s) = \#\{i : x_s^k = x_{s_i}^{k_i}\} - \#\{i : x_s^k = x_{s_i'}^{k_i'}\}$$

and

$$m(\sigma) = \sum_{s \in S} \sum_{k \in \mathcal{K} : d(\sigma,k,s) > 0} d(\sigma, k, s)$$

Characterization

e-Perturbed Strong Axiom for Revealed OEU

For any test sequence of pairs $(x_{s_i}^{k_i}, x_{s_i'}^{k_i'})_{i=1}^m \equiv \sigma$, we have

$$\prod_{i=1}^{m} \frac{\rho_{s_i}^{k_i}}{\rho_{s_i'}^{k_i'}} \leqslant (1+e)^{m(\sigma)}$$

Theorem

Let $e \in \mathbf{R}_+$, and D be a dataset. The following statements are equivalent:

- *D is e-perturbed OEU rational,*
- *D satisfies e-PSAROEU.*

Implementation

- Use e-price-perturbed OEU: for all $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1+e$$

- Problem:

$$\begin{aligned} \min_{(\varepsilon_s^k, v_s^k, \lambda^k)_{s,k}} \max_{k,s,t} \quad & \varepsilon_s^k / \varepsilon_t^k \\ \text{s.t.} \quad & \mu_s^* v_s^k = \lambda^k \varepsilon_s^k p_s^k \\ & x_s^k > x_t^{k'} \quad \Rightarrow \quad \log v_s^k \leqslant \log v_t^{k'} \end{aligned}$$

Implementation

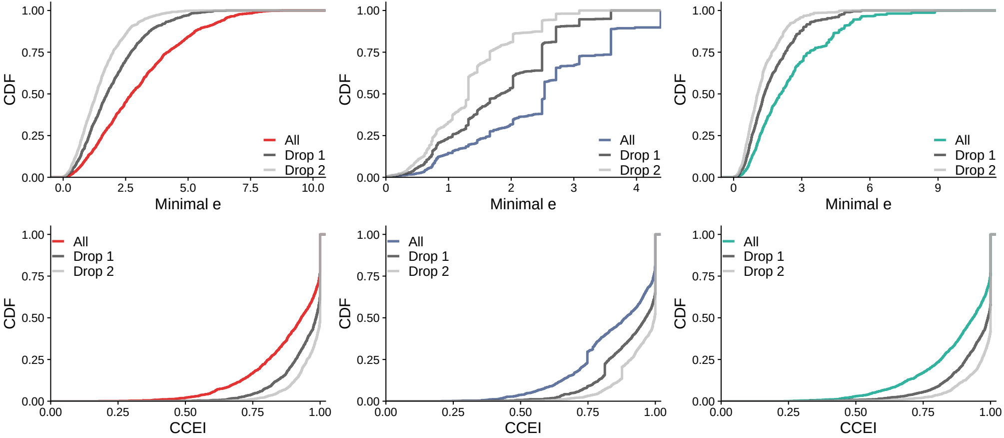
- Use e-price-perturbed OEU: for all $k \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k}{\varepsilon_t^k} \leqslant 1+e$$

- Problem:

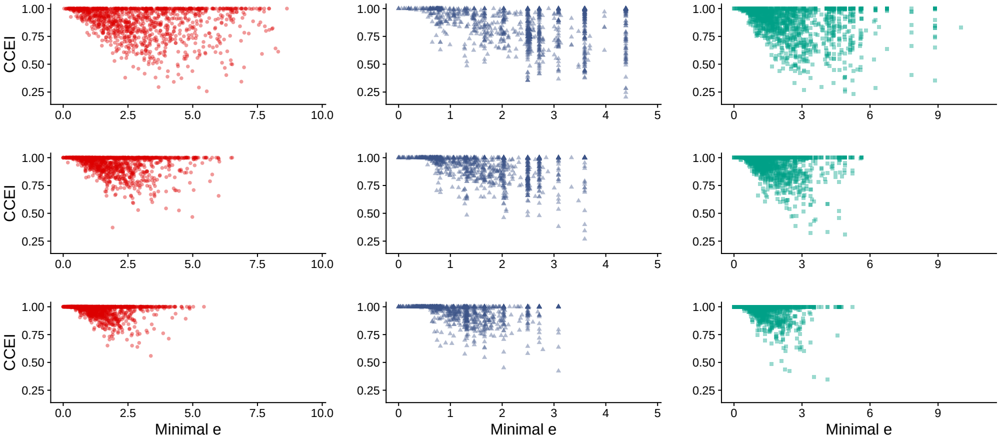
$$\begin{aligned} \min_{(v_s^k)_{s,k}} \quad \max_{k,s,t} \quad & (\log \mu_s^* + \log v_s^k - \log p_s^k) \\ & - (\log \mu_s^* + \log v_t^k - \log p_t^k) \\ \text{s.t.} \quad & x_s^k > x_t^{k'} \quad \Rightarrow \quad \log v_s^k \leqslant \log v_t^{k'} \end{aligned}$$

- Drop 1 or 2 “critical” mistakes
- Changes in distributions of e_* and CCEI



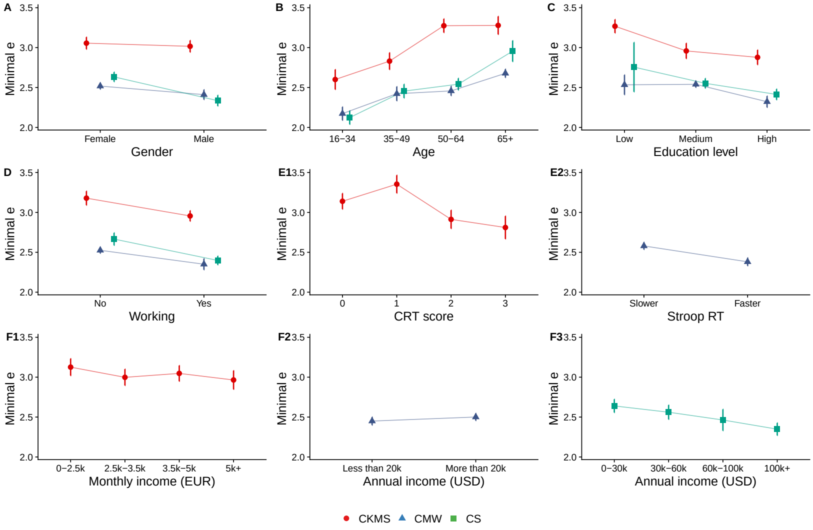
Sensitivity

- Drop 1 or 2 “critical” mistakes
- Correlation between e_* and CCEI

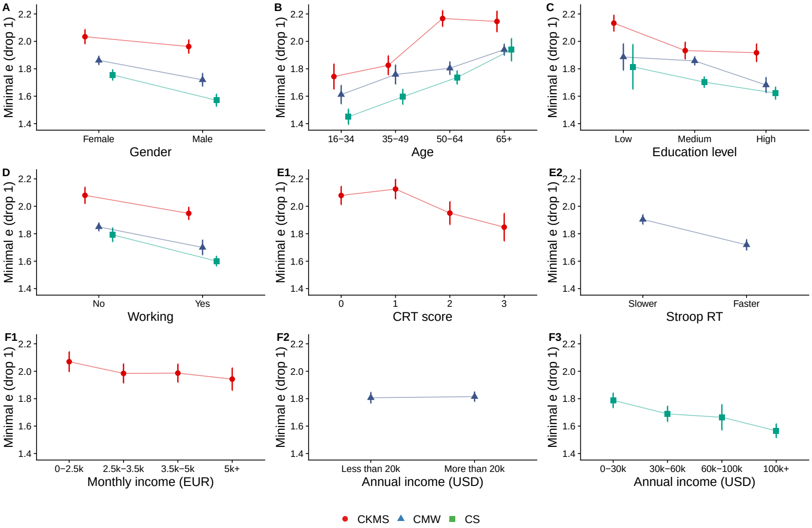


Sensitivity

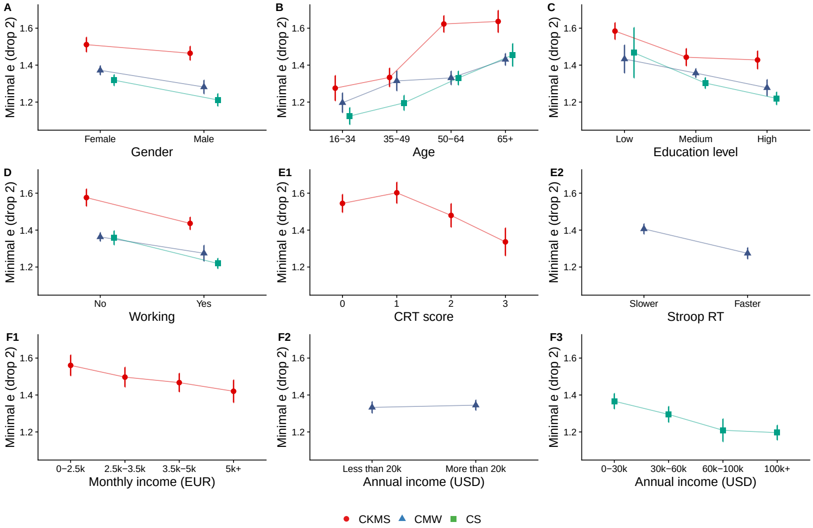
- Demographic variables (all 25 obs.)



- Demographic variables (drop 1 critical mistake)

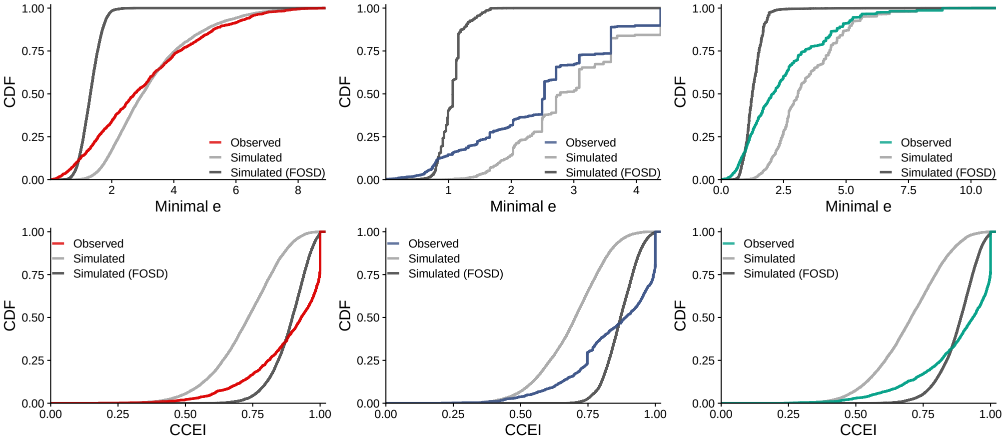


- Demographic variables (drop 2 critical mistakes)



Understanding the size

- Benchmark 1: Uniform random choice
- Benchmark 2: Uniform random choice respecting FOSD-monotonicity



Minimum perturbation test

- When can we say that e_* is large enough to reject consistency with OEU rationality allowing perturbation?

Procedure

- Price perturbation $\widetilde{p}_s^k = p_s^k \varepsilon_s^k$, $\varepsilon_s^k > 0$
- Randomly draw $(\varepsilon_s^k)_{s,k}$ from a distribution $\log \varepsilon_s^k \sim N(0, \sigma^2)$
- Calculate $\widehat{e} = \max_{k,s,t} \varepsilon_t^k / \varepsilon_s^k$
- Repeat $\rightsquigarrow$ empirical distribution of $\widehat{e}$
- Find the critical value $C_{0.05}$
- Reject the null if $e_* > C_{0.05}$

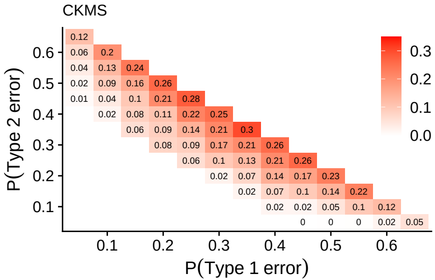
Minimum perturbation test

- What do we assume regarding the variance of ε?
- Our solution $\rightsquigarrow$ error ε as misperception
- Imagine a DM who mistakes "true" prices p and "perturbed" prices $\widetilde{p} = p\varepsilon$
  - true variance of p is σ_0^2
  - implied variance of $\widetilde{p}$ is $\sigma_1^2 > \sigma_0^2$
  - DM "tests" the null of $\sigma^2 = \sigma_0^2$ against the alternative of $\sigma^2 = \sigma_1^2 > \sigma_0^2$
  - We want σ_0^2 and σ_1^2 to be close (otherwise DM would not confuse p and $\widetilde{p}$)
  - We assume the sum of

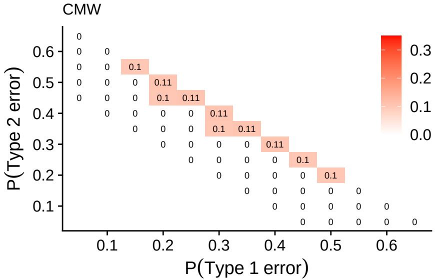
$$\eta^{I} = \Pr[\text{type } I \text{ error}]$$
$$\eta^{II} = \Pr[\text{type } II \text{ error}]$$

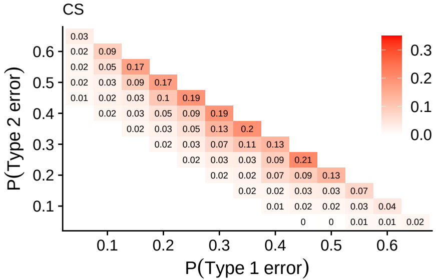
is relatively large

Minimum perturbation test



- For the largest value of $\eta' + \eta''$, at most 30% of subjects "significantly" violates OEU
- For most subjects, deviation from OEU could be attributed to relatively small mistakes





SEU rationalization

Definition

A dataset $(\boldsymbol{x}^k, \boldsymbol{p}^k)_{k=1}^K$ is *Subjective Expected Utility (SEU) rational* if there exist $\mu \in \Delta_{++}(S)$ and concave and strictly increasing $u : \mathbf{R}_+ \to \mathbf{R}$ such that for all $k \in \mathcal{K}$ and all $\boldsymbol{y} \in \mathbf{R}_+^{|S|}$,

$$\sum_{s \in S} p_s^k y_s \leqslant \sum_{s \in S} p_s^k x_s^k \implies \sum_{s \in S} \mu_s u(y_s) \leqslant \sum_{s \in S} \mu_s u(x_s^k)$$

- Interpretation: agent solves max $\sum \mu_s u(y_s)$ s.t. $\sum p_s^k y_s \leqslant I$ where $I = \sum p_s^k x_s^k$

e-Perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s u(x_s^k) \quad \text{s.t.} \quad \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \text{ for each observation } k$$

- Fix a number $e \in \mathbf{R}_+$
- e-belief-perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s^k u(x_s^k) \quad \text{s.t.} \quad \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k)$$

  - for each $k, l \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\mu_s^k / \mu_t^k}{\mu_s^l / \mu_t^l} \leqslant 1 + e$$

e-Perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s u(x_s^k) \quad \text{s.t.} \quad \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \text{ for each observation } k$$

- Fix a number $e \in \mathbf{R}_+$
- e-utility-perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s \left(u(x_s^k) \varepsilon_s^k \right) \quad \text{s.t.} \quad \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k)$$

  - for each $k, l \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k / \varepsilon_s^k}{\varepsilon_t^l / \varepsilon_t^l} \leqslant 1 + e$$

e-Perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s u(x_s^k) \;\; \text{s.t.} \;\; \boldsymbol{x}^k \in B(\boldsymbol{p}^k, \boldsymbol{p}^k \cdot \boldsymbol{x}^k) \text{ for each observation } k$$

- Fix a number $e \in \mathbf{R}_+$
- e-price-perturbed SEU

$$\max_{\boldsymbol{x}^k} \sum_{s \in S} \mu_s u(x_s^k) \;\; \text{s.t.} \;\; \boldsymbol{x}^k \in B(\widetilde{\boldsymbol{p}}^k, \widetilde{\boldsymbol{p}}^k \cdot \boldsymbol{x}^k), \quad \widetilde{p}_s^k = p_s^k \varepsilon_s^k$$

  - for each $k, l \in \mathcal{K}$ and $s, t \in S$

$$\frac{1}{1+e} \leqslant \frac{\varepsilon_s^k / \varepsilon_s^k}{\varepsilon_t^l / \varepsilon_t^l} \leqslant 1 + e$$

Characterizing e-perturbed SEU

e-Perturbed Strong Axiom for Revealed SEU

For any test sequence of pairs $(x^{k_i}_{s_i}, x^{k'_i}_{s'_i})^m_{i=1} \equiv \sigma$, if each s appears as s_i (on the left of the pair) the same number of times it appears as s'_i (on the right), then

$$\prod_{i=1}^{m} \frac{p^{k_i}_{s_i}}{p^{k'_i}_{s'_i}} \leqslant (1+e)^{m(\sigma)}$$

Theorem

Let $e \in \mathbf{R}_+$, and D be a dataset. The following statements are equivalent:

- *D is e-perturbed SEU rational,*
- *D satisfies e-PSARSEU.*