

Inspiring Regime Change

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February 20, 2022

Question

- citizens choose among a repertoire of **contentious performances** (Tilly 08):
 - public meetings, boycotts, strikes, marches, demonstrations, sit-ins, freedom rides, street blockades, suicide bombings, assassination, hijacking, and guerrilla war

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 - public meetings, boycotts, strikes, marches, demonstrations, sit-ins, freedom rides, street blockades, suicide bombings, assassination, hijacking, and guerrilla war
- but what contentious performances can be elicited from supporters and how?
 - **pleasure in agency** (Wood 03): “the positive effect associated with self-determination, autonomy, self-esteem, efficacy, and pride that come from the successful assertion of intention”
 - “depends on the likelihood of success, which in turn depends on the number participating... yet the pleasure in agency is undiminished by the fact that one’s own contribution to the likelihood of victory is vanishingly small”

Question

- **transformational leadership:** ability to create inspirational motivations via a variety of psychological mechanisms (Burns 78)
 - sociology: leaders raise participation by “identification, idealization, elevation of one or more values...” (Snow et al. 89)
 - politics: “people-oriented leaders are those who inspire people, give them a sense of identity...” (Goldstone 01)

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- **transformational leadership**: ability to create inspirational motivations via a variety of psychological mechanisms (Burns 78)
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What happens when **transformative leaders** manipulate **pleasure in agency** to influence activists' choice of **contentious performances** to maximize the likelihood of **regime change**?

1 Inspirational leadership as Mechanism Design:

Optimal Inspiration Entails Optimal Screening.

2 Organization of Anti-regime Movements:

When transformative leaders act optimally and strategically, revolutionary vanguard arise naturally. Consistent with Lenin's notion.

Distribution of anti-regime actions: regime change due to large efforts of a few, or due to smaller efforts of many?

Matters for the nature of post-revolutionary regime (Wantchekon and Garcia-Ponce 2017; Kadivar 2018).

More exogenous heterogeneity leads to less likelihood of regime change: within group vs between group inequality (Acemoglu and Robinson; Esteban and Ray).

Key Tradeoffs

- More optimistic citizens are easier to motivate.
- Higher benefits from intermediate levels of contentious performances (“effort”) will...
 - encourage those who would otherwise have chosen low effort to choose intermediate effort
 - discourage those who might otherwise have chosen high effort

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- optimism is endogenous and private information: derived from a coordination problem.

Model of Coordination

- **continuous action regime change game:**
- citizens make a continuous effort decision.
 - Efforts (**contentious performances**) are costly.
 - Rewards are success-contingent (**pleasure in agency**).
- revolution succeeds if aggregate effort exceeds a threshold
- small uncertainty about threshold \Rightarrow unique equilibrium (**global game of regime change**)

...and Screening

- leader can pick optimal reward scheme:
 - how **contentious performances** map into **pleasure in agency**
- We show that this becomes a **screening problem** because of (endogenously) heterogeneous beliefs about the probability of success

Methodological Contribution

To analyze a setting where coordination and screening are intertwined:

- **a screening problem arises because of the endogenous heterogeneity that is introduced by strategic uncertainty.**

We show that this problem can be disentangled into into:

- continuous action regime change game (Guimaraes Morris 2007)
- screening problem (Mookherjee and Png 1994)



Another Application: Threshold Public Goods

- Players decide how much to contribute to a project that succeeds whenever the aggregate contribution exceeds an uncertain threshold about which players have private information.
- The project manager or the fundraiser is then like the principal who seeks to maximize the likelihood of success by choosing recognition for donors of unknown types making heterogeneous contributions.

Model

- continuum of citizens with each choosing e_i .
- revolution succeeds if $\int e_i di \geq \theta$, where θ is “regime strength.”
 - θ is uncertain, and citizens have private signals about it.
- uncontingent cost of effort $C(e)$.
- contingent benefit of effort $B(e)$.
- optimal effort correspondence:

$$e^*(p) = \arg \max_{e \geq 0} pB(e) - C(e).$$

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The Spirit of '76



Liberty, Equality, Honor, Patriotism, Religious Sentiments

The Spirit of '76



Courage, honor, gallantry in service of liberty, all those words calculated to bring a blush of embarrassment to jaded twentieth-century men, defined manhood for the eighteenth century (Middlekauff 2005).

Exogenous Benefits and Complete Information

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- optimal effort correspondence:

$$e^*(p) = \arg \max_{e \geq 0} pB(e) - C(e).$$

- A correspondence is *weakly increasing* whenever:

$$p_2 > p_1 \Rightarrow e_2 \geq e_1, \forall e_i \in e^*(p_i), i = 1, 2.$$

Exogenous Benefits and Complete Information

- assume the optimal effort correspondence is increasing, so that $e^*(1) \geq e^*(0)$.
- maximum effort level is: $\bar{e} = \max(e^*(1))$.
- minimum effort level is: $\underline{e} = \min(e^*(0))$.

Equilibrium:

- 1 $\theta > \bar{e}$: everyone puts in $e^*(0)$ and the regime survives.
- 2 $\theta \leq \underline{e}$: everyone puts in $e^*(1)$ and there is a regime change.
- 3 $\underline{e} < \theta \leq \bar{e}$:
 - everyone puts in \underline{e} and the regime survives.
 - everyone puts in \bar{e} and there is a regime change. (lots of other eq)

Exogenous Benefits and Incomplete Information

- citizens receive private signals: $x_i = \theta + \nu_i$.
- recall that each citizen's problem is:

$$\max_{e_i \geq 0} pr(\text{success} | x_i) \times B(e_i) - C(e_i).$$

- strategy: $s_i(x_i) : \mathbb{R} \rightarrow \mathbb{R}_+$. Focus on decreasing strategies.
- maintain that $e^*(p)$ is increasing.

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- a strategy profile $(s_i)_{i \in [0,1]}$ gives rise to aggregate efforts:

$$\hat{s}(\theta) = \int_{i=0}^1 \int_{\nu_i=-\infty}^{\infty} s_i(\theta + \nu_i) f(\nu_i) d\nu_i di$$

Exogenous Benefits and Incomplete Information

- focusing on weakly decreasing strategies, $\hat{s}(\theta)$ is decreasing, and:
 - There is a unique θ^* such that $\hat{s}(\theta^*) = \theta^*$.
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$$s_i(x_i) = e^*(\text{pr}(\text{success} | x_i)) = e^*(\text{pr}(\theta < \theta^* | x_i)).$$

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$$\theta^* = \hat{s}(\theta^*) = \int e^*(\text{pr}(\theta < \theta^* | x_i)) d\mu(\text{pr}(\theta < \theta^* | x_i) | \theta^*).$$

Exogenous Benefits and Incomplete Information

A Statistical Property

- Suppose θ is an unknown and uncertain random variable with a uniform distribution—improper distribution on \mathbb{R} when relevant.
- Consider a signal $x = \theta + \nu$, where ν and θ are independent, and $\nu \sim f(\cdot)$.
- For a given threshold θ^* , what is $Pr(\theta < \theta^* | x)$?
- Because we have no prior information about x and θ , we can treat θ as a signal of x :

$$\theta = x - \nu.$$

- Thus,

$$Pr(\theta < \theta^* | x) = Pr(x - \nu < \theta^* | x) = Pr(x - \theta^* < \nu) = 1 - F(x - \theta^*).$$

A KEY Statistical Property

- At the equilibrium critical threshold θ^* , beliefs about the likelihood of regime change is distributed uniformly.

Let $H(\cdot|\theta^*)$ be the conditional CDF of beliefs given $\theta = \theta^*$.

$$\begin{aligned}
 H(p|\theta^*) &= Pr(\theta < \theta^* | x_i) \leq p | \theta^* \\
 &= Pr(1 - F(x_i - \theta^*) \leq p | \theta^*) \\
 &= Pr(\theta^* + F^{-1}(1 - p) \leq x_i | \theta^*) \\
 &= 1 - F(\theta^* + F^{-1}(1 - p) - \theta^*) \\
 &= 1 - (1 - p) = p.
 \end{aligned}$$

Exogenous Benefits and Incomplete Information

Recall that:

$$\theta^* = \hat{s}(\theta^*) = \int e^*(\text{pr}(\theta < \theta^* | x_i)) d\mu(\text{pr}(\theta < \theta^* | x_i) | \theta^*).$$

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$$\theta^* = \int_{p=0}^1 e^*(p) dp$$

An Example

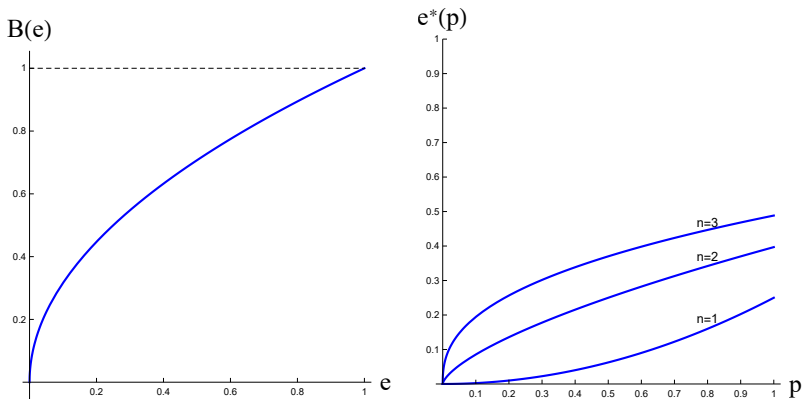


Figure 1: Optimal Effort for Exogenous Rewards. The exogenous reward scheme is $B(e) = \sqrt{e}$ and the exogenous punishment scheme is $C(e) = e^n$, $n \geq 1$. The right panel depicts $e^*(p)$ for $n = 1, 2, 3$.

The Road to Endogenous Benefits

We assumed that $e^*(p)$ is a weakly increasing correspondence in order to characterize the equilibrium:

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When is $e^*(p)$ is a weakly increasing? Recall optimal effort correspondence:

$$e^*(p) = \arg \max_{e \geq 0} p B(e) - C(e).$$

- If $C(e)$ is strictly increasing in e , then any selection from optimal effort correspondence is weakly increasing.

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- If $C(e)$ is strictly increasing in e , then any selection from optimal effort correspondence is weakly increasing. No matter what $B(e)$ is. Even when $B(e)$ is decreasing. Even when $B(e)$ is negative.

The Road to Endogenous Benefits

Let $p_2 > p_1 > 0$, $e_i \in e^*(p_i) = \arg \max_{e \geq 0} p_i B(e) - C(e)$, $i \in \{1, 2\}$.

Suppose $e_2 < e_1$.

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Suppose $e_2 < e_1$.

- Optimality of e_1 and e_2 :

$$p_2 B(e_2) - C(e_2) \geq p_2 B(e_1) - C(e_1) \Leftrightarrow p_2 [B(e_2) - B(e_1)] \geq C(e_2) - C(e_1)$$

$$p_1 B(e_1) - C(e_1) \geq p_1 B(e_2) - C(e_2) \Leftrightarrow C(e_2) - C(e_1) \geq p_1 [B(e_2) - B(e_1)]$$

Thus,

$$p_2 [B(e_2) - B(e_1)] \geq C(e_2) - C(e_1) \geq p_1 [B(e_2) - B(e_1)].$$

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Thus,

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- $C(e)$ is strictly increasing. Thus, $e_2 < e_1$ implies $C(e_2) < C(e_1)$.

$$0 > C(e_2) - C(e_1) \geq p_1 [B(e_2) - B(e_1)] \Rightarrow B(e_2) - B(e_1) < 0.$$

Endogenous Benefits

- Leader chooses success-contingent $B : \mathbb{R}_+ \rightarrow [0, M]$ to maximize the probability of regime change.
- Citizens observe $B(e)$ and their private signals and simultaneously decide how much effort to put in. Effort costs $C(e)$.
- Success or failure of the regime change attempt is realized.

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- Success or failure of the regime change attempt is realized.
- Assume $C(e)$ is strictly increasing and convex, with $C(0) = 0$.

$$\begin{aligned} & \max_{B(\cdot)} \int_{p=0}^1 e^*(p) dp \\ & \text{s.t. } e^*(p) = \arg \max_{e \geq 0} p B(e) - C(e) \\ & \quad B(e) \in [0, M], \end{aligned}$$

Inspirational Leadership As Mechanism Design

$$\begin{aligned} \max_{B(\cdot)} \int_0^1 e^*(p) dp \\ B(e) \in [0, M] \\ e^*(p) = \arg \max_{e \geq 0} pB(e) - C(e) \end{aligned}$$

- **As if** the leader gives a citizen psychological rewards $B(e)$ in exchange for revolutionary contributions e .
 - A citizen's valuation of these rewards p is her private information.
- **Screening:** More optimistic citizens are easier to motivate, but a citizen's optimism is her private information.
 - If $C(e)$ is non-linear, this is MD with non-transferable utility and non-standard budget (links to money burning literature?).
- **Screening+Coordination:** These valuations are endogenously determined in a coordination interaction.

Screening Problem: Linear Cost Benchmark

$$\max_{B(\cdot)} \int_0^1 e^*(p) dp$$

$$B(e) \in [0, M]$$

$$e^*(p) = \arg \max_{e \geq 0} pB(e) - ce$$

- Suppose optimal reward scheme is a step function: a citizen gets M if $e \geq \hat{e}$, and 0 otherwise. Then, sufficiently optimistic citizens exert \hat{e} and others don't contribute: $pM \geq c\hat{e}$. $\theta^* = (1 - c\hat{e}/M)\hat{e}$.
- Can we restrict attention to step function reward schemes?
- A monopolist is selling a single unit to buyers with valuations $v \sim U[0, M/c]$.
- Type- p buyer with valuation pM/c makes a payment $e^*(p)$ and receives the item with probability $B(e^*(p))/M$. The seller's revenue is $\int_0^1 e^*(p) dp$. This is Riley and Zackhauser (1983).
 - IC: $e^*(p) = \arg \max_{e \geq 0} pB(e)/c - e$.

Screening Problem

- revelation principle / optimal screening argument:
- for each type $p \in [0, 1]$, a “contract” is offered: $(e(p), B(p))$.

$$\max_{\{(e(p), B(p))\}} \int_{p=0}^1 e(p) dp$$

$$s.t. \quad pB(p) - C(e(p)) \geq 0, \quad \forall p \in [0, 1]$$

$$pB(p) - C(e(p)) \geq p B(p') - C(e(p')), \quad \forall p, p' \in [0, 1]$$

$$B(p) \in [0, M], \quad \forall p \in [0, 1].$$

Screening Problem: Incentive Compatibility

$$pB(p) - C(e(p)) \geq p B(p') - C(e(p'))$$

$$p' B(p') - C(e(p')) \geq p' B(p) - C(e(p))$$

$$\Rightarrow (p - p') (B(p) - B(p')) \geq 0.$$

$\Rightarrow B(p)$ is weakly increasing

$\Rightarrow B(p)$ is almost everywhere differentiable. Let $h(p) = C(e(p))$.

\Rightarrow IC becomes $p B'(p) - h'(p) = 0$ (FOC) and $B'(p) \geq 0$ (SOC).

Screening Problem

$$\max_{\{(e(p), B(p))\}} \int_{p=0}^1 e(p) dp$$

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$$B(p) \in [0, M], \quad \forall p \in [0, 1].$$

$$\max_{\{(e(p), B(p))\}} \int_{p=0}^1 e(p) dp$$

$$s.t. \quad pB'(p) - h'(p) = 0, \quad B'(p) \geq 0, \quad h(0) = 0$$

$$B(p) \in [0, M].$$

Screening Problem

- Recall that $h(p) = C(e(p))$, and hence $e(p) = C^{-1}(h(p))$. Letting $\Pi(\cdot) = C^{-1}(\cdot)$, we have:

$$\max_{\{(h(p), B(p))\}} \int_{p=0}^1 \Pi(h(p)) dp$$

$$s.t. \quad pB'(p) - h'(p) = 0, \quad B'(p) \geq 0, \quad h(0) = 0$$

$$B(p) \in [0, M].$$

- If $\Pi(h(p)) \neq h(p)$, we have a mechanism design with non-transferable utility: the “agent” gives up $h(p)$, the “principal” gets $\Pi(h(p))$.

A Lemma

Lemma: If the optimal effort correspondence e^* is single-valued, continuous and weakly increasing, and $e_{min} = 0$, then

$$\theta_{e^*} = \int_{p=0}^1 e^*(p) dp = \int_{e=0}^{e_{max}} \left(1 - \frac{C'(e)}{B'(e)} \right) de.$$

Proof: (1) $pB'(e^*(p)) = C'(e^*(p))$. (2) Integration by parts:

$$\int_{p=p_1}^{p_2} e^*(p) dp = p_2 e^*(p_2) - p_1 e^*(p_1) - \int_{e=e^*(p_1)}^{e^*(p_2)} \frac{C'(e)}{B'(e)} de.$$

Main Result

An optimal reward function $e^*(p)$ is continuous and increasing with $e^*(0) = 0$. Then, we show that the leader's problem is to choose (\bar{e}, B') to maximize

$$\int_0^{\bar{e}} \left(1 - \frac{C'(e)}{B'(e)} \right) de$$

subject to the marginal benefit constraint that

$$B(\bar{e}) = \int_{e=0}^{\bar{e}} B'(e) de = M.$$

Thus, the leader's problem becomes a constrained point-wise optimization. The Lagrangian is

$$\begin{aligned} L &= \int_0^{\bar{e}} \left(1 - \frac{C'(e)}{B'(e)} \right) de + \lambda \left(M - \int_{e=0}^{\bar{e}} B'(e) de \right) \\ &= \int_0^{\bar{e}} \left(1 - \frac{C'(e)}{B'(e)} - \lambda B'(e) \right) de + \lambda M. \end{aligned}$$

Local Tradeoffs

$$L = \int_0^{\bar{e}} \left(1 - \frac{C'(e)}{B'(e)} - \lambda B'(e) \right) de + \lambda M.$$

- Point-wise maximization w.r.t. $B'(e)$:

$$MB = \frac{C'(e)}{(B'(e))^2} \text{ and } MC = \lambda.$$

- Point-wise maximization w.r.t. \bar{e} :

$$MB = 1 - \frac{C'(\bar{e})}{B'(\bar{e})} \text{ and } MC = \lambda B'(\bar{e}).$$

Inspirational Leadership: Analytical Solutions

- The equilibrium regime change is

$$\theta^* = \bar{e} - \frac{M}{4} \frac{1}{C'(\bar{e})}.$$

- The maximum, endogenous revolutionary effort \bar{e} is the unique solution to

$$\sqrt{C'(\bar{e})} \int_0^{\bar{e}} \sqrt{C'(x)} dx = M/2.$$

- The strictly increasing segment of the optimal effort function has a simple characterization

$$e^*(p) = C'^{-1}((2p)^2 C'(\bar{e})).$$

Main Result

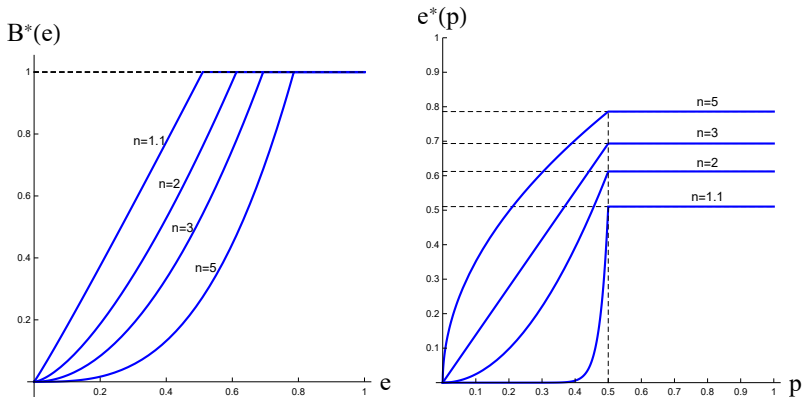


Figure 2: Optimal reward schedule, $B^*(e)$, and its induced effort schedule, $e^*(p)$, for $M = 1$ and cost functions of the form $C(e) = e^n$, $n > 1$. As the right panel illustrates, when the cost function approached the linear $C(e) = e$, effort schedule becomes binary as described in the text.

Recall $B(e) = \sqrt{e}$

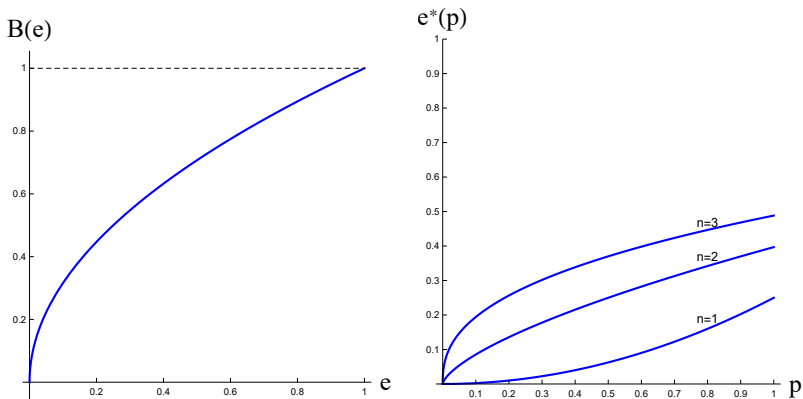


Figure 3: Optimal Effort for Exogenous Rewards. The exogenous reward scheme is $B(e) = \sqrt{e}$ and the exogenous punishment scheme is $C(e) = e^n$, $n \geq 1$. The right panel depicts $e^*(p)$ for $n = 1, 2, 3$.

Inspiration in the American Revolution

- **Washington to the militia:** take up your musket and join us “for a few weeks, perhaps only a few days”
- **Washington to the inhabitant of the Middle Colonies:** give us stock cattle to feed the army. If not, at least don't give it to the British.
- **Paine's** first essay of *The American Crisis*: “these are the times that try men's souls.”
- Abraham **Keteltas's** sermon, *God Pleads His Cause*.
- **Washington to the Congress:** “The honor of making a brave defense does not seem to be a sufficient stimulus, when the success is very doubtful and the falling into the Enemy's hands probable”
- **the loyalist Nicholas Cresswell of Virginia in his diary** after the battle of Trenton: “The minds of the people are much altered. A few days ago they had given up the cause for lost. Their late successes have turned the scale and now they are all liberty mad again.”

Distribution of Revolutionary Effort

- If regime change is mainly the result of high effort by a narrow subset of the population, the post-revolution regime is likely to reflect the views of that narrow group.
- mass mobilization → more enduring democracy (Kadivar 2018).
- anti-colonial movements associated with rural insurgencies increase the chances of autocracies in post-colonial regimes (García-Ponce and Wantchékon 2017).

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- Equilibrium distribution of beliefs given a state: $H(p|\theta)$.
- Optimal effort function: $e^*(p)$.
- Equilibrium distribution of efforts given a state: $J(e|\theta)$.

Distribution of Revolutionary Effort

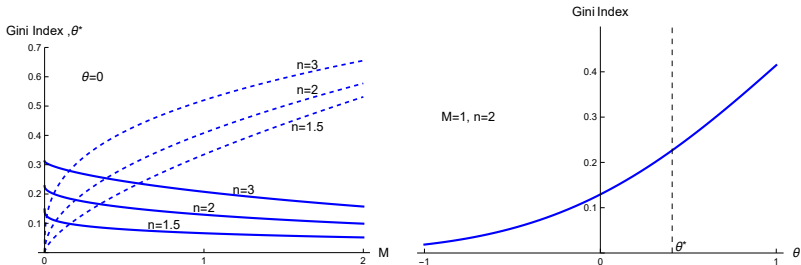


Figure 4: Left: Gini Index for the distribution of revolutionary effort (solid curves) and θ^* (dashed curves) as functions of M , when $\theta = 0$, $C(e) = e^n$, $n = 1.5, 2, 3$, and $F = N(0, 1)$. Right: Gini Index as a function of θ when $M = 1$, $C(e) = e^2$, and $F = N(0, 1)$.

Distribution of Revolutionary Effort

- Two countries, A and B , with $\theta_A > \theta_B$. In country A , $\theta_A = \theta^* + 0.1$, so that the regime survives absent, e.g., a drop in the regime's strength. In country B , $\theta_B = \theta^* - 0.1$, so that there will be a regime change. $GI(\theta_A) > GI(\theta_B)$.
- Now, suppose after citizens decide their contributions in country A , the regime's strength suddenly drops from θ_A to θ_B , e.g., due to the unexpected decision of armed forces to remain neutral or to switch sides as happened in Iran in 1979 (Amanat 2017) or in Egypt in 2011 (Barany 2011).
- Regime changes that occur as a result of an unexpected decision of the military to switch sides also tend to generate less democratic post-revolution regimes.

Exogenous Heterogeneity

In addition to having different information, citizen differ in their propensity to be influence because of their **past interactions with the regime, economic and social status, moral convictions:**

$$p_i \alpha_i B(e_i) - C(e_i), \text{ with } \alpha \sim F_\alpha.$$

- If x_i and α_i are independent, still $p_i \sim U[0, 1]$ in equilibrium.
- As if there is an augmented type: $t = p \times \alpha \sim G$:

$$\theta^* = \int e^*(t) dG(t).$$

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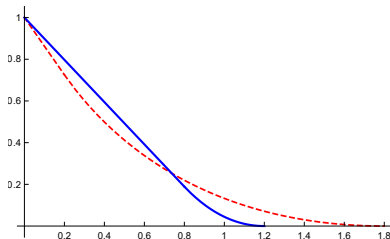
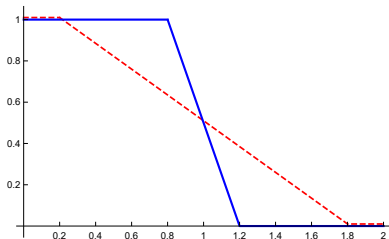
- As if there is an augmented type: $t = p \times \alpha \sim G$, where $\alpha \sim F_\alpha$ and $p \sim U[0, 1]$.
- If $C(e) = e$, optimal screening will be a posted price mechanism:
 - Get M if and only if $e > \hat{e}$ for some optimally chosen \hat{e} .
 - Exert \hat{e} if and only if $tM > \hat{e}$

$$\max_{\hat{e}} \theta^*(\hat{e}) = \max_{\hat{e}} (1 - G(\hat{e}/M)) \hat{e}.$$

Exogenous Heterogeneity: Demand Rotation

Left graph: $G = U[1 - \delta, 1 + \delta]$, with $\delta \in (0, 1)$.

Right graph: $\alpha \sim U[1 - \delta, 1 + \delta]$, $p \sim U[0, 1]$, $t = p \times \alpha \sim G$.



When the leader's optimal strategy is to seek the participation of a majority of potential participants a mean-preserving spread lowers $\theta^*(\delta)$ (The last step is: Johnson and Myatt 2006).

Exogenous Heterogeneity: Demand Rotation

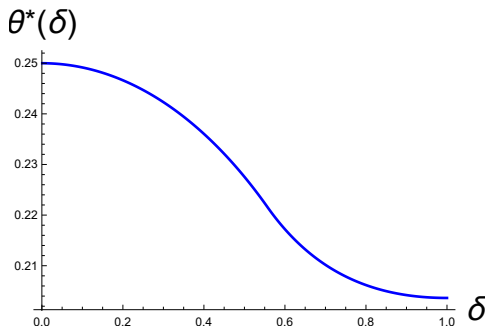


Figure 5: Equilibrium regime change threshold as function of the dispersion δ of exogenous heterogeneity. $C(e) = e$, $M = 1$, and $F_\alpha = U[1 - \delta, 1 + \delta]$.

Optimal Contribution Restrictions

- Suppose $B(e)$ is exogenous and strictly concave, but the leader/manager can restrict efforts $e \geq 0$ to $\{e_1, \dots, e_N\}$ for some $N \geq 1$.
- Whether and when he would restrict contributions? To how many and what contribution levels?

RESULT. Under some assumptions that ensures $e^*(p)$ is a strictly increasing function:

- If $C(e)$ is strictly concave, then the leader is strictly better off if he can restrict contributions to a single level.
- If $C(e)$ is linear, then he is indifferent.
- If $C(e)$ is strictly convex, then he is strictly better off not intervening.

The Curious One Half

- Suppose $C(e) = e$, so that $h'(p) = C'(e) e'(p) = e'(p)$.
- Suppose types are distributed like $f(p)$.

$$\max_{\{(e(p), B(p))\}} \int_{p=0}^1 e(p) f(p) dp$$

$$s.t. \quad pB'(p) - e'(p) = 0, \quad B'(p) \geq 0, \quad e(0) = 0$$

$$B(p) \in [0, M].$$

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$$B(p) \in [0, M].$$

$$e(p) = \int_{x=0}^p e'(x) dx.$$

The Curious One Half

$$\begin{aligned} e(p) &= \int_{x=0}^p e'(x) dx = \int_{x=0}^p xB'(x) dx = xB(x) \Big|_{x=0}^p - \int_{x=0}^p B(x) dx \\ &= pB(p) - \int_{x=0}^p B(x) dx. \end{aligned}$$

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 \end{aligned}$$

Moreover,

$$\begin{aligned}
 \int_{p=0}^1 \left(\int_{x=0}^p B(x) dx \right) f(p) dp &= \left(\int_{x=0}^p B(x) dx \right) F(p) \Big|_{p=0}^1 - \int_{p=0}^1 B(p) F(p) dp \\
 &= \int_{p=0}^1 (1 - F(p)) B(p) dp.
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 &= \int_{p=0}^1 (1 - F(p)) B(p) dp.
 \end{aligned}$$

$$\theta^* = \int_{p=0}^1 e(p) f(p) dp = \int_{p=0}^1 \left(p - \frac{1 - F(p)}{f(p)} \right) B(p) f(p) dp.$$

The Curious One Half

$$\max_{B(p) \in [0, M]} \int_{p=0}^1 \left(p - \frac{1 - F(p)}{f(p)} \right) B(p) f(p) dp.$$

$$B^*(p) = \begin{cases} 0 & ; p - \frac{1 - F(p)}{f(p)} < 0 \\ M & ; p - \frac{1 - F(p)}{f(p)} > 0. \end{cases}$$

- Uniform Case: $p - \frac{1 - F(p)}{f(p)} = p - \frac{1 - p}{1} = 0 \Rightarrow p = 1/2.$