Three Layers of Uncertainty

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Motivation (1)

Uncertainty is pervasive and plays a major role in economics.

Deep uncertainty/ambiguity is more and more recognized to play a central role in decision-making processes

- Partly because of some recent "catastrophic" events
 - Economic uncertainty: financial crisis
 - Technological uncertainty: Fukushima
 - Health uncertainty: COVID-19
- Partly because of a growing awareness about
 - Environmental uncertainty: climate change
 - Demographic uncertainty: longevity / mortality risk

Motivation (2)

The notion of ambiguity (deep uncertainty/Knightian uncertainty)

"all kind of situations in which a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem she is facing" (Cerreia-Vioglio et al., 2013)

Motivation (3)

Examples in climate change economics (Heal and Millner, 2014)

Sources of ambiguity:

- scientific: incomplete understanding of the climate system (GHG emissions changes in temperatures)
- socio-economic: impacts climate change can have on our socio-economic environment and response of human societies

IPCC:

"In most instances, objective probabilities are difficult to estimate. Furthermore, a number of climate change impacts involve health, biodiversity, and future generations, and the value of changes in these assets is difficult to capture fully in estimates of economic costs and benefits."

Motivation (4)

- Ambiguity attitudes might play an important role in economic problems
 - *Robust* optimal monetary policy rule may lead to amplification in the response of the optimal policy to shocks (Giannoni, 2002)
 - AA may affect treatment decisions in the health domain (Berger et al, 2013)
 - AA can have notable effects on climate change policies (Millner et al. 2013, Drouet et al. 2015, Berger et al. 2017)

The project

Objective

 Better understand individual behavior in the face of ambiguity/uncertainty

How?

 $\rightarrow\,$ decompose ambiguity into layers

Why it matters?

- Important for the construction of realistic models capable of making accurate predictions
- Important for prescriptive applications guiding decision-making processes

Road map

- 2. Three layers of uncertainty
- 3. Experimental design
- 4. Uncertainty Premia and Predictions
- 5. Results
- 6. Robustness
- 7. Conclusion

2. Three layers of uncertainty

Three layers of uncertainty

(All uncertainty relevant for decision making is ultimately subjective.)

Yet, in applications (especially with data) it is convenient to distinguish between different layers of uncertainty.

Following Arrow (1951), Hansen (2014), Marinacci (2015), Hansen and Marinacci (2016), we decompose ambiguity into 3 distinct layers:

- 1. Risk (aleatory uncertainty)
- 2. Model ambiguity (epistemic uncertainty)
- 3. Model misspecification (epistemic uncertainty)

1. **Risk** (= aleatory uncertainty, physical uncertainty)

- Situations with an objectively known probability distribution



- uncertainty about states: variability within a particular probability model
- examples: chance mechanisms (roulette, coin, dice)
- deals with variability in data (because of inherent randomness, measurement errors, omitted minor explanatory variables)
- characterizes data generating processes (DGP) (i.e. probability models)
- probability is an objective measure of randomness/variability

2. model ambiguity (= epistemic uncertainty)

- Arises when then DM is not able to identify a single probability model (among a given set) corresponding to the phenomenon of interest



• uncertainty *across* [structured] models

- ex: deals with the truth of propositions
 - "the composition of the urn is P% red and 1 P% black balls"
 - "the parameter that characterizes the DGP has value x"
- Notation: $M = \{m \in \Delta(S) : p \text{ s.t. } p \in [0, 1]\},$ or, here: $M = \{P\%, Q\%\}$
- epistemic uncertainty may be quantified by means of *subjective* probabilities
- ightarrow probability=measure of degree of belief

3. Model misspecification (= epistemic uncertainty)

- Arises when the set of models under consideration might not include the correct model



- uncertainty *about* models
- in real-life problems, models are, by design, approximations (= simplification of complex phenomena)
- \rightarrow The set *M* is *misspecified*
- emerges as the result of the approximate nature of the models under consideration
- this layer of uncertainty has also an *epistemic* nature

Useful framework to analyze decision problems under ambiguity

 \Rightarrow These three layers are inherent in any decision problem under uncertainty where the DM has probabilistic theories about the outcomes of a phenomenon and forms beliefs over their relevance

Example 1: Ellsberg two-color urn with 100 balls

- 101 models (physical compositions)
- $M = \{m_{ heta} = rac{ heta}{100} ext{ for } heta \in \{0, 1, ..., 100\}\}$
- no misspecification by construction



Useful framework to analyze decision problems under ambiguity

Example 2: estimates of the TCR

The transient climate response: how much the planet will immediately warm once we reach the level of doubled CO_2



- different models exists
- which is the "correct" one?
- 3 layers together

Useful framework to analyze decision problems under ambiguity

Example 3: When different experts provide opinions about the probability of an event (e.g., developing a disease, fire risk in buildings, aircraft accidents, climate catastrophe, etc.)

 \rightarrow Expert judgments on the risk of collapse of the Atlantic Meridional Overturning Circulation (AMOC) due to global climate change



- 12 leading climate scientists (observationalists, palaeoclimatologists, modelers)
- large scale impacts: strong cooling by several degrees, increase in sea level up to 1m (direct) + shift of the Intetropical Convergence Zone, warming of the Southern ocean (indirect)

This paper

- Empirical investigation of the three layers of uncertainty
- First examination of model misspecification in a laboratory environment
 - Previous research typically uses standard Ellsberg (1961) paradigm
 - Hence: no misspecification by construction
- \rightarrow New *extended* Ellsberg setting:
 - 1. leave the number of possible compositions unspecified
 - 2. change the information about possible compositions
 - 3. isolate the effects of model ambiguity and model misspecification
 - 3 experiments: university students + risk professionals + online

Related literature

- Multi-stage presentation of uncertainty and its relation to ambiguity
 - Empirical: Halevy (2007), Chew et al. (2017), Abdellaoui et al. (2015), Armentier and Treich (2016), Chew et al. (2018), Berger and Bosetti (2020)
 - Theoretical: Segal (1987; 1990), Klibanoff et al. (2005), Nau (2006), Seo (2009)
- Describe uncertainty preferences of individuals with different backgrounds
 - general population (Dimmock et al. 2015, 2016), children and adolescents (Sutter et al. 2013), business owners (Viscussi and Chesson, 1999),
 - actuaries (Hogarth and Kunreuther, 1989; Cabantous, 2007; Cabantous et al. 2011)

3. Experimental Design

General

Experimental Design

- Within-subject design
- Individual choices under different sources of uncertainty
- Sources may encompass different layers
- Three experiments with the same design:
 - Iab with students
 - lab-in-the-field with risk professionals
 - online (with students)
- Real money incentives

Experimental Design: 5 sources

Simple risk

 $M = \{50\%\}$

Compound risk (x2)

- 1. $M = \{25\%, 75\%\}$
- 2. $M = \{0\%, 100\%\}$

Model ambiguity (x2)

- 1. $M = \{25\%, 75\%\}$
- 2. $M = \{0\%, 100\%\}$

Model misspecification (x2)

1. $M = \{25\%, 75\%\}$ 2. $M = \{0\%, 100\%\}$

Ellsberg Ambiguity (x2)

1. M = [0, 1]2. $M = \{0\%, 1\%, ..., 100\%\}$

Experimental Design: sources

Simple risk

 $M = \{50\%\}$

Ellsberg Ambiguity (x2)



Experimental Design: sources

Compound risk (x2)

- 1. $M = \{25\%, 75\%\}$

- Ellsberg Ambiguity (x2)

Compound risk 1

Receive $\in 20$ with either 75% probability or 25% probability, each with equal likelihoods



Experimental Design: sources

Compound risk (x2) 2. $M = \{0\%, 100\%\}$ Ellsberg Ambiguity (x2)

Compound risk 2

Receive $\in 20$ with either 100% probability or 0% probability, each with equal likelihoods



Experimental Design: sources

Model ambiguity (x2) 1. $M = \{25\%, 75\%\}$ Ellsberg Ambiguity (x2)

Model ambiguity 1

Receive $\in 20$ with either 75% probability or 25% probability



Experimental Design: sources

Model ambiguity (x2)

- 2. $M = \{0\%, 100\%\}$

Ellsberg Ambiguity (x2)

Model ambiguity 2

Receive $\in 20$ with either 100% probability or 0% probability



Experimental Design: sources

Simple risk

Model misspecification (x2)

1. $M = \{25\%, 75\%\}$ Ellsberg Ambiguity (x2)

Model misspecification 1

Receive $\in 20$ with either 75% probability or 25% probability or possibly another probability



Experimental Design: sources

Model misspecification (x2)

2. $M = \{0\%, 100\%\}$

Ellsberg Ambiguity (x2)

Model misspecification 2

Receive $\in 20$ with either 100% probability or 0% probability or possibly another probability



Experimental Design: sources

Simple risk Ellsberg Ambiguity (x2) 1. M = [0, 1]

Ellsberg Ambiguity 1

Receive €20 with an unknown probability, and €0 otherwise



Experimental Design: sources



Ellsberg Ambiguity 2 Receive €20 with an unknown probability, and $\in 0$ otherwise ?%___•€20 ?%``•€0 100 cards... ?% ?%

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Elicitation of Certainty Equivalents: Choice List Design

50%		1	50%		
(€20 with 50%	(€20 with 50% probability) ~ €.				
Option 1			Option 2		
Have €20 if the card drawn is black	\odot		Have €0 for sure		
Have €20 if the card drawn is black	0	0	Have €1 for sure		
Have €20 if the card drawn is black			Have €3 for sure		
Have €20 if the card drawn is black	0	0	Have €5 for sure		
Have €20 if the card drawn is black	\bigcirc		Have €7 for sure		
Have €20 if the card drawn is black	0	0	Have €9 for sure		
Have €20 if the card drawn is black	0		Have €11 for sure		
Have €20 if the card drawn is black	0	0	Have €13 for sure		
Have €20 if the card drawn is black			Have €15 for sure		
Have €20 if the card drawn is black	0	0	Have €17 for sure		
Have €20 if the card drawn is black			Have €19 for sure		
Have €20 if the card drawn is black	0	0	Have €20 for sure		

Elicitation of Certainty Equivalents: Choice List Design

50%		50%		
(€20 with 50% probability) ~ $\in X$?				
Option 1		Option 2		
Have €20 if the card drawn is black	• •	Have €0 for sure		
Have €20 if the card drawn is black	• •	Have €1 for sure		
Have €20 if the card drawn is black	• •	Have €3 for sure		
Have €20 if the card drawn is black	0 00	Have €5 for sure		
Have €20 if the card drawn is black	0	Have €7 for sure	-	
Have €20 if the card drawn is black	0	Have €9 for sure	5+7	
Have €20 if the card drawn is black	•	Have €11 for sure	$LE = \frac{1}{2} = 6$	
Have €20 if the card drawn is black	0	Have €13 for sure	<u> </u>	
Have €20 if the card drawn is black	•	Have €15 for sure		
Have €20 if the card drawn is black	0	Have €17 for sure		
Have €20 if the card drawn is black	0	Have €19 for sure		
Have €20 if the card drawn is black	0	Have €20 for sure		

Design summary

- 9 Certainty Equivalents
 - Simple Risk (SR)
 - Compound Risk {0,100} (CR0)
 - Compound Risk {25,75} (CR25)
 - Model ambiguity {0,100} (MA0)
 - Model ambiguity {25,75} (MA25)
 - Model Misspecification {0, 100} (MM0)
 - Model Misspecification {25,75} (MM25)
 - Extended Ellsberg Ambiguity [0, 100] (EE)
 - Standard Ellsberg Ambiguity $\{0, 1, ..., 100\}$ (SE)

Procedure

Main experiment

- 125 Bocconi students (average age: 20.5; 42% female)
- Bets yield €20 or €0
- Within-subject random incentives, (one choice question picked prior to the experiment to be played for real at the end, Johnson et al. 2021)
- Participation fee: €5

4. Uncertainty Premia and Predictions

Uncertainty Premium (1)

Definition (1)

The *total uncertainty premium* Π_i is defined as

$$\exists_i \equiv EV_i - CE_i$$

for all $i \in \{SR, CR0, CR25, MA0, MA25, MM0, MM25, EE\}$.

 \rightarrow amount of money that an individual is willing to pay to receive the expected value of the prospect with certainty, rather than facing the uncertainty.

 $\rightarrow \Pi_i$ is >0 (resp. =0, or <0) when a subject is averse (resp. neutral, or loving) to the uncertainty in prospect *i*.

 \rightarrow Most well-known absolute uncertainty premium is the standard risk premium: Π_{SR}

(1)

Uncertainty Premium (2)

Definition (2) The *differential uncertainty premium* $\Pi_{i,j}$ is defined as

$$\Pi_{i,j}\equiv \Pi_j-\Pi_i$$

for all $i, j \in \{SR, CR0, CR25, MA0, MA25, MM0, MM25, EE\}$.

Here:

$$\Pi_{i,j} = CE_i - CE_j$$

 \rightarrow difference between the CE of the bet on prospect i and the CE of the bet on prospect j

 \rightarrow Ex 1: $\Pi_{SR,j}$ is the compound risk premium when $j \in \{CR\}$

 \rightarrow Ex 2: $\Pi_{SR,j}$ is the *ambiguity premium* when $j \in \{MA, MM, EE\}$

 \rightarrow Def 1: $\Pi_{CR,MA}$ is the model ambiguity premium

 \rightarrow Def 2: $\Pi_{\textit{MA},\textit{MM}}$ is the model misspecification premium

(2)

Predictions

EU hypothesis

$$\Pi_{SR,CR} = \Pi_{SR,MA} = \Pi_{SR,MM} = 0 \tag{3}$$

• ambiguity and compound risk neutrality

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Layer hypothesis

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$$\begin{cases} \Pi_{SR,CR} = 0 \\ \Pi_{CR,MA} \neq 0 \text{ or } \Pi_{MA,MM} \neq 0 \text{ (or both).} \end{cases}$$
(4)

- Distinction between layers of uncertainty
- In line with Klibanoff et al. (2005), Gilboa and Schmeidler (1989), ...

Predictions - cont

Stage hypothesis

$$\begin{cases} \Pi_{SR,CR} \neq 0 \\ \Pi_{CR,MA} = \Pi_{MA,MM} = 0. \end{cases}$$

- Distinction between stages of uncertainty only
- Violation of the reduction of compound risk axiom
- In line with Segal (1989), Seo (2009)

Stage and layer hypothesis

$$\begin{cases} \Pi_{SR,CR} \neq 0 \\ \Pi_{CR,MA} \neq 0 \text{ or } \Pi_{MA,MM} \neq 0 \text{ (or both).} \end{cases}$$

- Distinction between stages and layers of uncertainty
- In line with Ergin and Gul (2009)

(6)

(5)

5. Results (students)

Data quality

- Discard data with multiple-SW, reverse-SW or no-SW patterns
 - 3.6% for students
 - Significantly lower than typical 10% observed in the literature (Yu et al. 2020)

5.1. Attitudes towards uncertainty

4.0 -3.5 3.0 fotal uncertainty premium 2.5 2.0 - \times SR O P = 0 $\triangle P = 25$ 1.5 E EE 1.0 0.5 0.0 -0.5 SR FF ĊR MM MA Source of Uncertainty

Total uncertainty premium Π_i

- Simple risk neutrality (p=0.16)
- Π_i > 0 for the other sources (CR0, p=0.099)

5.1. General attitudes towards uncertainty

Differential uncertainty premium $\Pi_{i,j}$

	P = 0	P = 25	Average
Π _{SR,CR}	$0.10 \ (N = 118)$	1.40*** (N = 117)	0.74*** (<i>N</i> = 120)
Π _{SR,MA}	1.18^{***} ($N = 119$)	1.81^{***} ($N = 117$)	1.46*** (<i>N</i> = 120)
Π _{SR,MM}	1.71^{***} ($N = 119$)	2.10^{***} ($N = 120$)	1.91^{***} (N = 120)
Π _{SR,EE}			2.30*** (N = 116)

Notes: The number of observations is in parentheses. Average premia are based on all non-missing values. ***p-value<0.01, **p-value<0.05, *p-value<0.1, based on two-sided t-tests.

Ambiguity aversion

• Compound risk aversion when P = 25, but not when P = 0 (p=0.58)

5.2. Decomposing attitudes towards uncertainty

Differential uncertainty premium $\Pi_{i,j}$

	P = 0	P = 25	Average
П _{CR,MA}	1.11^{***} (N = 119)	0.50*** (N = 116)	0.84*** (<i>N</i> = 120)
П _{МА, MM}	0.50** (N = 120)	$0.37^* (N = 118)$	0.53*** (<i>N</i> = 122)
П _{СR,MM}	1.48 ^{***} (<i>N</i> = 120)	0.67^{***} ($N = 118$)	1.05*** (N = 122)
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Notes: The number of observations is in parentheses. Average premia are based on all non-missing values. ***p-value<0.01, **p-value<0.05, *p-value<0.1, based on two-sided t-tests.

- Distinction between CR and MA: $\Pi_{CR,MA} > 0$
- $\rightarrow\,$ subjects are ready to pay on average 8.4% of their expected payoff to avoid the layer of model ambiguity
 - Distinction between MA and MM: $\Pi_{MA0,MM0} > 0$
- $\rightarrow\,$ subjects are ready to pay an extra 5.3% to avoid the layer of model misspecification

5.3 Explaining the uncertainty premium: the role of layers

Relevance of distinguishing layers while explaining the overall uncertainty premium

Regression analysis of total premia with subject fixed effects

- M1 (EU): $\Pi_i^s = \alpha_0 + \gamma^s + \varepsilon_i^s$
- M2 (Stages): $\Pi_i^s = \beta_0 + \beta_1 TS0_i + \beta_2 TS25_i + \gamma^s + \varepsilon_i^s$
- M1' (Layers): ; $\Pi_i^s = \alpha_0 + \alpha_1 M A_i + \alpha_2 M M_i + \gamma^s + \varepsilon_i^s$
- M2' (S&L): $\Pi_i^s = \beta_0 + \beta_1 TS0_i + \beta_2 TS25_i + \beta_3 MA_i + \beta_4 MM_i + \gamma^s + \varepsilon_i^s$

(Π_i^s is the total uncertainty premium for prospect *i* for subject *s*, γ^s is the individual fixed effect, TSO_i and $TS25_i$ are dummies for prospects presented in two stages, MA_i and MM_i are dummies for prospects entailing the 2nd and 3rd layers)

5.3 Explaining the uncertainty premium: the role of layers

	No distinction between stages		Distinctio	Distinction between stages	
	Model 1	Model 1'	Model 2	Model 2'	
	(EU)	(Layers)	(Stages)	(Stages & layers)	
MA		1.003***		0.758***	
MM		1.413***		1.166***	
<i>TS</i> 0			1.001***	0.357**	
<i>TS</i> 25			1.760***	1.113***	
Constant	1.644***	0.952***	0.462***	0.464***	
Observations	845	845	845	845	
Notes: *** p-value	Notes: *** p -value < 0.01 ** p -value < 0.05 * p -value < 0.1				

- Introduction of layers or stages increases the uncertainty premium
- Reject EU and Stage hypotheses (layer of risk only, F -test, p<0.001)
 - Reject hypothesis of no distinction between MA and model MM (p=0.015 and p=0.016)
- Stages also matter (p < 0.001).

5.4. Individual-level analysis

Classify subjects according to the predictions

EU hyp.	Layer hyp.	Stage hyp.	Stage and layer hyp.
19.7%	31.2%	8.1%	41.0%

- Most common preference pattern (41%) is consistent with the hybrid stage and layer hypotheses (non-reduction of CR & distinct attitudes towards layers)
- 2nd most common pattern (31%) is in line with the layer hypothesis (reduction of CR & non-neutrality towards ambiguity)
- 20% are consistent with the EU hypothesis (neutrality towards CR and ambiguity)
- Pure stage hypothesis holds for 8% of the subjects.

6. Robustness: Follow-up experiments

Potential concerns:

- Artifacts of the potential limitations of the subject pool to deal with complexity of our sources? → result in an aversion towards sources with several stages and layers of uncertainty
- Consequence of order effects or contagion between sources encompassing distinct layers (from successive evaluation in the within-subjects design)?

Procedure

Risk professionals

- 84 subjects from 33 countries (average age:~40; 44% female)
- Bets yield €200 or €0
- Highly educated:
 - 69% MA, 21% PhD,
 - in the field of mathematics/statistics (55%), actuarial sciences (20%), physics, engineering, ...
- average 13 years experience in the insurance/finance industries
- Between-subject random incentives, PRINCE (Johnson et al. 2021)

Online platform (Prolific)

- Between-subject design
- 740 students, 18–35 y.o.
- ullet Bets yield £20 or £0, higher resolution in the choice list
- Between-subject random incentives, PRINCE
- Participation fee: £2 (for 16 min)

6.1. Attitudes towards uncertainty



7. Summary and Conclusion

Summary and Conclusion

- Simple experimental environment to study the three layers of uncertainty
- We demonstrate that there exists an empirical distinction between attitudes toward the three layers
 - Difficulty to address model misspecification in the lab
 - In reality, potential model misspecification is much more complicated
- Sophistication of the individuals matters for stages, not for layers