Macroeconomic Effects of Delayed Capital Liquidation

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Dec 2021

Capital reallocation with financial shocks

- Two types of capital reallocation (after which, in general, new productivity applies):
  - full liquidation (i.e., acquisition, about 70%);
  - partial liquidation (i.e., sales of properties, plants, and equipments, about 30%).
- In 2018, $0.81 trillion capital reallocation from COMPUSTAT non-financial firms:
  - about 32% of all capital expenditures;
  - the reallocation - expenditure ratio (R-E) is procyclical;
  - the partial liquidation share in total reallocation (P share) is countercyclical.
- Note: debt is also procyclical; how do financial shocks affect liquidation decisions, productivity, and output?

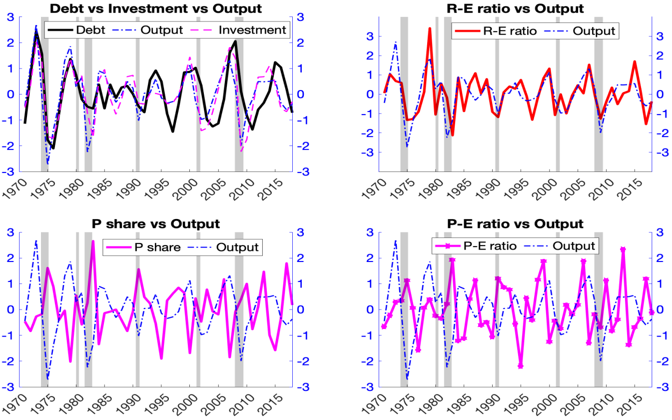
Contribution

- A theory of "financially-constrained option value" to understand liquidation decisions.
  - Firms face idiosyncratic liquidation cost and idiosyncratic productivity risks (well established facts).
- A threshold of liquidation cost; unproductive firms will
  - avoid possibly financial constraints if liquidated;
  - but they give up possible future smaller liquidation cost and/or productivity gain.
- After credit tightening, more unproductive firms will likely to stay in the medium term (if they can survive). Reasons:
  - lower debt-servicing cost;
  - GE effects, i.e., lower wages and lower interest rates;
  - the quantitative exercises assess the importance of financial shocks.
- Note: productivity shocks produce the opposite, cleansing effect!

Literature

- Capital reallocation: Jovanovic - Rousseau (2002), Eisfeldt and Rampini (2006), Cao and Shi (2016), Wright, Xiao, and Zhu (2017), ...
- Financing constraints and partial irreversibility: Caggese (2007), Kahn and Thomas (2013), Lanteri (2018), ...
- Financial shocks: Jermann - Quadrini (2012), Del Negro et al (2017), ...
- This paper: a theory of leveraged option value (unproductive firms may exhibit higher firm leverage)
  - together with the aggregate implication of financial shocks.

Procyclical debt, investment, and overall reallocation



...but countercyclical P share and P-E ratio.

Summary statistics

Corr.	Debt	R-E ratio	P-E ratio	P share	Output
Debt	1	0.52 (0.60)	-0.15 (-0.38)	-0.45 (-0.62)	0.59 (0.79)
R-E ratio	-	1	-0.16 (-0.30)	-0.77 (-0.86)	0.64 (0.66)
P-E ratio	-	-	1	0.75 (0.75)	-0.17 (-0.39)
P share	-	-	-	1	-0.53 (-0.66)
Output	-	-	-	-	1
Rel. Std. Dev.	1.19	5.79	5.59	8.68	1

Note: Numbers in brackets are the corresponding correlations for NBER recessions.
Note: The correlations of investment with output, the R-E ratio, the P share are 0.85, 0.57, and -0.49, respectively.

A model of financially constrained option value

- Why might low productive firms, whose capital not being liquidated, be financially constrained?
- After a tightening of credit, are they more or less likely to liquidate capital?

A firm problem

- An entrepreneur with preference

$$u(c) = \log(c).$$

  - c can be interpreted as dividends.
  - The preference can also be used to modeling dividend smoothing.
- If running a firm with capital k, the gross return is $R^k = r + 1 - \delta$ where $r \geq 0$ and $\delta \in [0, 1]$.
- Risk-free bonds with return R.

Resale and financial frictions

- *Resale frictions*- when selling, have to sell the whole firm (note: this assumption will be relaxed)

$$k_{t+1} \in \{0\} \cup [(1-\delta)k_t, +\infty).$$

- i.i.d. *stochastic utility liquidation cost* $\zeta \in [\underline{\zeta}, \bar{\zeta}]$ with a CDF $F(.)$.
  - ζ sometimes drives the entrepreneur to liquidate;
  - other times it forces them to stay in business.
- *Financial frictions*- collateral constraints

$$Rb_{t+1} \geq -\theta(1-\delta)k_{t+1},$$

and θ measures the tightness.

The entrepreneur's problem

$$V(k,b,\zeta) = \max\{V^0(k,b) - 1_{\{k>0\}}\zeta, \quad V^1(k,b)\}.$$

where if not running business

$$V^0(k,b) = \max_{c,b_{+1}} \{u(c) + \beta \mathbb{E}[V(0,b_{+1},\zeta_{+1})]\} \text{ s.t.} \tag{1}$$

$$c + b_{+1} = R^k k + Rb; \tag{2}$$

$$b_{+1} \geq 0, \tag{3}$$

if running business

$$V^1(k,b) = \max_{c,k_{+1},b_{+1}} \{u(c) + \beta \mathbb{E}[V(k_{+1},b_{+1},\zeta_{+1})]\} \text{ s.t.} \tag{4}$$

$$c + b_{+1} + k_{+1} = R^k k + Rb; \tag{5}$$

$$Rb_{+1} \geq -\theta k_{+1}; \tag{6}$$

$$k_{+1} - (1-\delta)k \geq 0. \tag{7}$$

Policy function

Proposition

Define $N^0(k,b) \equiv R^k k + Rb$ *and* $N^1(k,b) \equiv rk + q\left(\frac{k}{k+b}\right)(1-\delta)k + Rb$ *as net worths. Then,*

$$V^0(k,b) = J^0 + \frac{\log N^0(k,b)}{1-\beta}, V^1(k,b) = J^1\left(\frac{k}{k+b}\right) + \frac{\log N^1(k,b)}{1-\beta},$$

where J^0 *is a constant and where* $J^1(\lambda)$ *and* $q(\lambda) \le 1$ *are functions of leverage* $\lambda \equiv k/(k+b)$. *Further,* $q < 1$ *means that the resale constraint is strictly binding. The consumption, capital, and bond policy functions have the following algebraic forms:*

$$c = \begin{cases} (1-\beta)N^0 & \textit{not running} \\ (1-\beta)N^1 & \textit{running} \end{cases}; \quad k_{+1} = \begin{cases} 0 & \textit{not running} \\ \frac{\lambda_{+1}\beta N^1}{1+(q-1)\lambda_{+1}} & \textit{running} \end{cases};$$

$$b_{+1} = \begin{cases} \beta N^0 & \textit{not running} \\ \frac{(1-\lambda_{+1})\beta N^1}{1+(q-1)\lambda_{+1}} & \textit{running} \end{cases}.$$

Scale-invariant property

- The proposition says: consumes $1-\beta$ of net worths and saves β fraction...and we can focus on $k=1$ and leverage λ.
- Let $n^0(\lambda) = N^0(1, \lambda^{-1}-1)$ where

$$\lambda \leq \bar{\lambda} = \left(1 - \frac{\theta}{R}\right)^{-1}.$$

- "Scale-invariant" property: $\forall \rho > 0$

$$V(\rho k, \rho b, \zeta) = V(k, b, \zeta) + \frac{\log \rho}{1-\beta}. \tag{8}$$

- A liquidation threshold for a leverage λ
  - directly related to, but more useful than, option value.
- Comparing the value of liquidating and the value of staying.

A firm-abandoning problem

- To focus on liquidation, assume the firm is unproductive

$$R^k \equiv r + 1 - \delta < R. \tag{A1}$$

- For those firms not liquidated, $k_{+1} = (1-\delta)k$, and therefore

$$c + b_{+1} = rk + Rb.$$

- As the firm is unproductive, i.e., r is small, firm debt $-b_{+1}$ may need to be large or hit the financing constraint (6)
  - ...because of the smoothing need represented by $u(c) = \log(c)$.
  - The point is more general: any non-flexible cost will produce similar results.

Liquidation threshold

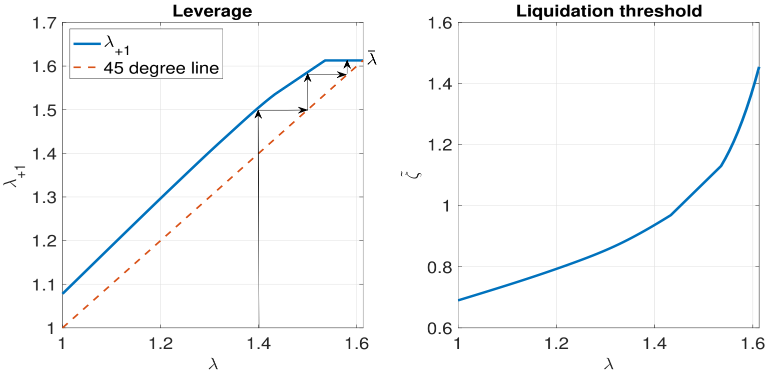
Proposition

Under some conditions including A1, the liquidation threshold satisfies the forward looking condition

$$\tilde{\zeta}(\lambda) = \log\left(\frac{(1-\beta)n^0(\lambda)}{n^0(\lambda) - (1-\delta)/\lambda_{+1}}\right) + \frac{\beta}{1-\beta}\log\left(\frac{R}{n^0(\lambda_{+1})}\right) + \frac{\beta}{1-\beta}\log\left(\frac{\beta n^0(\lambda)}{1-\delta}\right) + \beta\left[\tilde{\zeta}(\lambda_{+1}) - \int_{\underline{\zeta}}^{\tilde{\zeta}(\lambda_{+1})} F(x)dx\right].$$

- 1st term: difference between utilities of consumption today from liquidating and from staying;
- 2nd and 3rd terms: difference between continuation values;
- 4th term: recursive because of a similar liquidation decision next period.

Leverage dynamics and liquidation policy



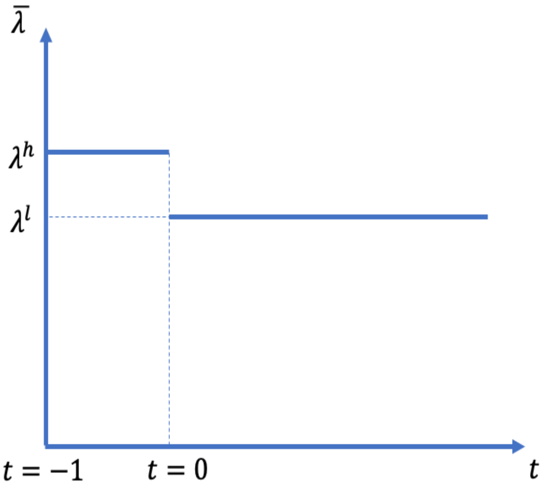
The threshold is an increasing function of leverage.
The incentive to liquidate is higher given a higher leverage (that implies a smaller c).

A thought experiment

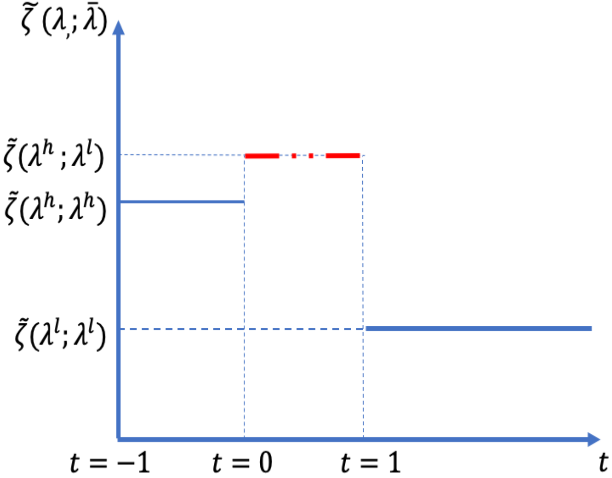
Suppose $\bar{\lambda}$ is fixed at a certain level λ^h until $t = 0$.

- Unexpectedly, $\bar{\lambda}$ falls permanently from λ^h to λ^l, from $t = 0$ onward.
- Consider a sample path of no liquidation.
- Focus on the case in which both resale and financial constraints are binding.
- Exclude forced liquidation.

A permanent tightening in credit



Liquidation threshold after shocks



What have we learned?

- Liquidation more likely on impact, but less likely in the medium term.
  - A tightened constraint makes the short run more painful;
  - However, it can raise the option value because of lower debt servicing cost (after forced deleveraging).
- When r is not too small, option value dominates and entrepreneurs are less likely to liquidate:
  - the liquidation threshold cost falls;
  - the probability of being liquidated falls.
- In the paper, if interest rate is endogenized, liquidation likelihood can fall also on impact.
- Warning: abstracts from forced liquidation!

The macro model with quantitative analysis

- Introducing GE effects (via interest and wage rates) that further delay capital liquidation.
- Quantitative assessment of the reallocation channel.

The representative household

A Representative Household

$$W(B^H; X) = \max_{C^H, L^H, B^H_{+1}} \left\{ U\left(C^H, L^H\right) + \beta^H \mathbb{E}\left[W(B^H_{+1}; X_{+1}) | X\right]\right\} \text{ s.t.}$$

$$C^H + B^H_{+1} = wL^H + RB^H, \tag{9}$$

where C^H is consumption, L^H is labor supply, B^H is bond holding, w is the wage rate, and X is the aggregate state.

Entrepreneurs

- Productivity z follows a Markov process ($z \in \{z^l, z^h\}$ with $0 < z^l < z^h$)

$$\mathrm{P}\{z_t = z^j \mid z_{t-1} = z^i\} = p^{ij},$$

  where $i, j \in \{l, h\}$.
- Production

$$y(i) = [z(i)k(i)]^{\alpha} [A\ell(i)]^{1-\alpha}.$$

- Profit rate is endogenous because

$$\Pi(z, k; w) = \max_{\ell} \left\{ (zk)^{\alpha}(A\ell)^{1-\alpha} - w\ell \right\} = \pi zk$$

  and

$$\pi = \alpha \left(\frac{(1-\alpha)A}{w} \right)^{\frac{1-\alpha}{\alpha}}.$$

Entrepreneurs (continued)

- Collateral constraints (with d as the resale discount):

$$R_{+1}b_{+1} \geq -\theta(z)(1-d)k_{+1}.$$

- Partially sell up to a ϕ fraction of existing capital:

$$k_{+1} \geq (1-\phi)(1-\delta)k.$$

- To get rid of R (and only use R_{+}), define $\tilde{\lambda}$

$$\tilde{\lambda} \equiv \frac{k}{k+Rb} \text{ and, thus,} \lambda = \frac{\tilde{\lambda}}{\tilde{\lambda}+(1-\tilde{\lambda})/R}.$$

- Define $\tilde{B} \equiv RB$ (bonds held by entrepreneurs) and $\tilde{B}^H \equiv RB^H$ (bonds held by households).

Market clearing

- Backward looking wealth dynamics:

$$K^h_{+1} = \frac{\tilde{\lambda}^h_{+1}}{\tilde{\lambda}^h_{+1} + (1 - \tilde{\lambda}^h_{+1})/R_{+1}} \beta \left[\sum_j \left(z^h \pi - \delta + \frac{1}{\tilde{\lambda}^j} \right) p^{jh} K^j + p^{lh} \tilde{B} \right];$$

$$K^l_{+1} = (1-\phi)(1-\delta) \sum_j \left[1 - F(\tilde{\zeta}^j) \right] p^{jl} K^j;$$

$$\tilde{B}_{+1}/R_{+1} = \beta \sum_j F(\tilde{\zeta}^j) \left(z^l \pi - \delta + \frac{1}{\tilde{\lambda}^j} \right) p^{jl} K^j + \beta p^{ll} \tilde{B}.$$

- Markets for credit and labor

$$\sum_j \left(\frac{1}{\tilde{\lambda}^j_{+1}} - 1 \right) K^j_{+1} + \tilde{B}_{+1} + \tilde{B}^H_{+1} = 0;$$

$$A^{-1} \left(\frac{\pi}{\alpha} \right)^{\frac{1}{1-\alpha}} \sum_j \left(p^{jh} z^h + p^{jl} z^l \right) K^j = L^H.$$

- See paper for equilibrium definition with optimization problems.

Some specification

- Productivity

$$\log(\tilde{z}^h) = \log\left(z^h\right)^{\alpha} = \sigma \text{ and } \log(\tilde{z}^l) = \log(z^l)^{\alpha} = -\sigma.$$

- Utility function

$$U(C_H, L_H) = \frac{\left(C_H - \frac{\mu L_H^{1+\nu}}{1+\nu}\right)^{1-\varepsilon} - 1}{1-\varepsilon}.$$

- Targeting R-E ratio and output volatility, by using exogenous shocks

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A;$$
$$\log \theta_t = (1-\rho_\theta)\log\theta + \rho_\theta \log\theta_{t-1} + \epsilon_t^\theta,$$

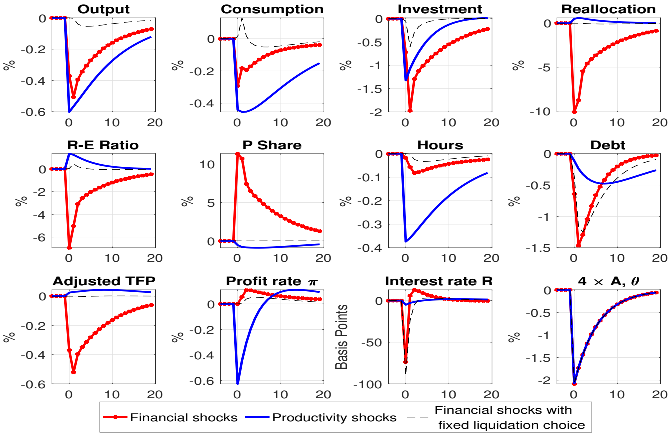
where $0 < \rho_A, \rho_\theta < 1$, $\epsilon_t^A \sim N(0, \sigma_A^2)$, and $\epsilon_t^\theta \sim N(0, \sigma_\theta^2)$ are i.i.d. normal.

Calibration

	Value	Explanation/Target		Value	Explanation/Target
β^H	0.98	Risk-free rate 2%	ϕ	3.96%	Share of partial sales: 28%
ε	1	Household risk aversion	δ	8.88%	Effective depr. rate: 10%
ν	1/1.5	Inv. labor supply elast.	$\bar{\zeta}$	10.54	See the discussion in text.
μ	2.34	Hours worked 1/3	$\underline{\zeta}$	3.559	R-E ratio: 30%
p^{hh}	0.845	Prod. persistence 0.69	ξ	34.49	Acq costs: 1.68% of output
p^{ll}	0.845	$p^{ll} = p^{hh}$	θ	0.42	Debt-to-output: 65.5%
$\tilde{z}^h$	1.28	Prod. std. dev. 0.18	m	0.10	Exogenous
$\tilde{z}^l$	0.78	$\log(\tilde{z}_l) = -\log(\tilde{z}_h)$	ρ_A	0.83	Exogenous
α	0.30	Capital share	ρ_θ	0.83	Exogenous
d	0.10	10% cost of partial sells	σ_A	0.52%	Output volatility 1.92%
β	0.90	Investment/output: 18.1%	σ_θ	2.09%	Relative R-E volatility 5.79

Impulse response functions

TFP shocks: cleansing effect; countercyclical R-E.
Financial shocks: procyclical R-E, investment, debt and countercyclical P share as in the data.



Business-cycle statistics

Corr.	Debt	R-E	P-E	P share	Output
Debt	1	0.52 (0.54)	-0.15 (-0.77)	-0.45 (-0.67)	0.59 (0.51)
R-E	-	1	-0.16 (-0.87)	-0.77 (-0.97)	0.64 (0.64)
P-E	-	-	1	0.75 (0.96)	-0.17 (-0.87)
P share	-	-	-	1	-0.53 (-0.78)
Output	-	-	-	-	1
Rel. Std. Dev	1.19 (1.29)	5.79 (5.79)	5.59 (5.00)	8.68 (10.43)	1

Note: Numbers in brackets are results from the model after I feed the smoothed shocks into the model.

Note: The correlations of investment with output, the R-E ratio, and the P share are 0.85 (0.95), 0.57 (0.56), -0.49 (-0.74), respectively.

Understanding the option value effect

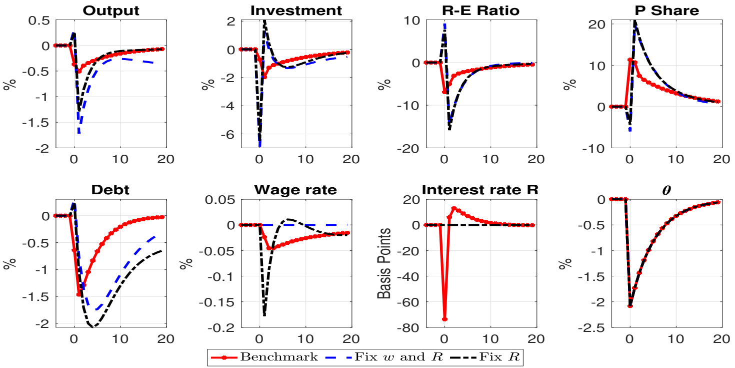
- Reallocation of used capital is thus persistently delayed.
- The magnitude is *not* mainly caused by the lower demand from productive firms.
- To understand this claim, recall

$$\text{R-E ratio} = \frac{L}{L+I} = \frac{FL+PL}{FL+PL+I}.$$

- To generate a falling L-E ratio, capital liquidation (L) must fall more than the fall of investment (I).
- Demand effect will only move L and I with the same amount.

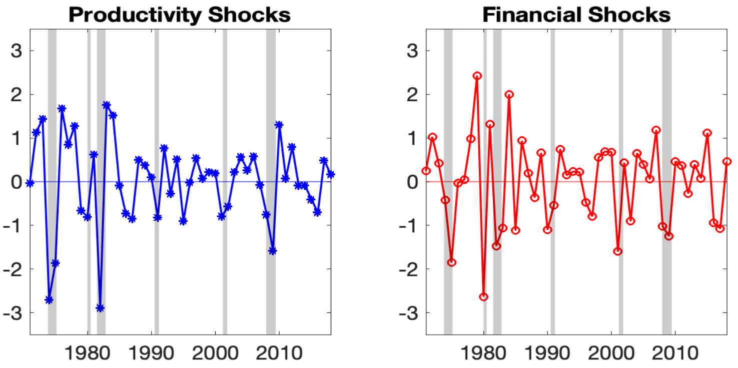
Counterfactuals

The GE effects are crucial in the quantitative analysis to obtain the co-movements of investment and reallocation.
A lower interest/wage rate makes staying option more attractive.



Smoothed shocks

Note: our analysis excludes the financial sector and housing issues; but the analysis still suggests that financial shocks have become relatively more important.



Conclusion

Takeaways

- A theory of financially-constrained option value of staying.
- The trade-off between staying and liquidating
  - which may imply a negative relationship between leverage and productivity (supported by the data).
- A tightened financing constraint can worsen the trade-off:
  - longer delay of liquidating an unproductive firm;
  - persistent worse capital allocation that endogenous reduce TFP;
  - new investment falls as well;
  - interest and wage rates amplify the effect and are crucial for the co-movement of new and old capital
- Implication for interest rate policy / capital tax policy.