Behavioral Influence

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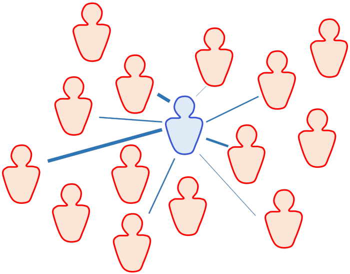
Decision Theory

Decisions are made in isolation!!!

In reality:

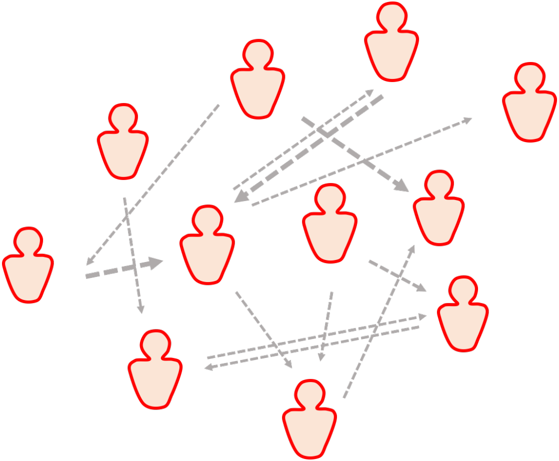


- People sharing the same environment such as members of the same household, friends, colleagues, neighbors, etc.
- We influence each other's behavior through advice, inspiration, imitation, etc.

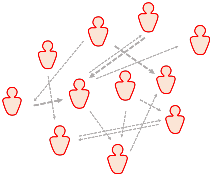
Social Interactions

- Huge (econometrical) literature on understanding the extent of social interactions in individual decisions:
  - productivity at work (Mas and Moretti, 2009)
  - job search (Topa, 2001)
  - school-achievement (Calvo-Armengol, et al., 2009)
  - teen smoking/drinking, recreational activities (Sacerdote, 2011)
  - adolescent pregnancy (Case and Katz, 1991)
  - crime (Glaser et al. 1996)

Identifying Network



Our Aim



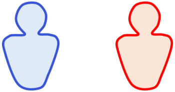
- Propose a choice-theoretic approach to social influence
  - Describe a simple model of interacting individuals
  - Detect influence from observed choice behavior
  - Quantify Influence and Identify Preference
  - Minimal Data

Road Map

1. Baseline Model: Two individuals, conformity behavior (positive)
2. General Model: Multi-individual interactions
3. Extension: Any type of influence (positive and/or negative)

Primitive

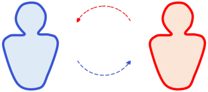
- Domain: $|X| > 1$ finite set of alternatives
- Two individuals: 1 and 2



- Data: $p_1(x, S)$ and $p_2(x, S)$, where

$$
\begin{aligned}
p_i(x,S) &> 0 \text{ for all } x \in S \\
\sum_{x \in S} p_i(x,S) &= 1
\end{aligned}
$$

Model



$$\text{choices} \equiv f(\text{individual component}, \text{choices of other})$$

$$p_1 \equiv f(w_1, p_2)$$

Model

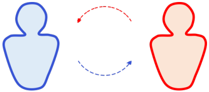
$$w_1(x) + \alpha_1 p_2(x, S)$$

- α_1 influence parameter for individual 1

$$p_1(x, S) = \frac{w_1(x) + \alpha_1 p_2(x, S)}{\sum_{y \in S} [w_1(y) + \alpha_1 p_2(y, S)]}$$

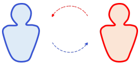
Isolation vs Society

$$p_1(x,S) = \frac{w_1(x)}{\sum\limits_{y \in S} w_1(y)}$$



$$p_1(x,S) = \frac{w_1(x) + \alpha_1 p_2(x,S)}{\sum\limits_{y \in S} [w_1(y) + \alpha_1 p_2(y,S)]}$$

A Hypothetical Example



- Two colleagues, Dan and Bob,
- Daily exercise routines during the pandemic
  - exercise home or
  - go for a walk outside.

	Dan	Bob
walk outside	0.71	0.78
exercise home	0.29	0.22

A Hypothetical Example

	Dan	Bob
walk outside	0.71	0.78
exercise home	0.29	0.22

- Two Possible Explanations
  - No influence and individual preferences are aligned
  - Individual preferences are not aligned but a strong influence
- Reflection Problem (Manski, 1993)

A Hypothetical Example

- Gyms are open NOW!!!

	Dan	Bob		Dan	Bob
walk outside	0.71	0.78		0.60	0.70
exercise home	0.29	0.22		0.26	0.19
go to the gym				0.14	0.11

- Observe that $\frac{0.71}{0.29} \approx 2.5 \neq 2.3 \approx \frac{0.60}{0.26}$
- !!!Existence of Influence!!!

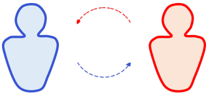
A Hypothetical Example

- Gyms are open NOW!!!

	Dan	Bob		Dan	Bob
walk outside	0.71	0.78		0.60	0.70
exercise home	0.29	0.22		0.26	0.19
go to the gym				0.14	0.11

- !!!Existence of Influence!!!
- We can *uniquely* identify
  - Dan and Bob have opposite rankings
  - Dan is strongly influenced by Bob

Model



$$p_1(x, S) = \frac{w_1(x) + \alpha_1 p_2(x, S)}{\sum_{y \in S}[w_1(y) + \alpha_1 p_2(y, S)]}$$

$$p_2(x, S) = \frac{w_2(x) + \alpha_2 p_1(x, S)}{\sum_{y \in S}[w_2(y) + \alpha_2 p_1(y, S)]}$$

Comment # 1

$$p_i(x,S) = \frac{w_i(x) + \alpha_i p_j(x,S)}{\sum_{y \in S} [w_i(y) + \alpha_i p_j(y,S)]}$$

Alternatively, we can express the model:

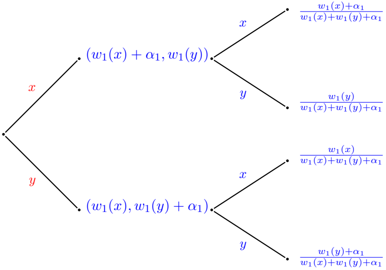
$$p_i(x,S) = \frac{\mu_i w_i(x) + (1-\mu_i) p_j(x,S)}{\sum_{y \in S} [\mu_i w_i(y) + (1-\mu_i) p_j(y,S)]}$$

where

$$\mu_i = \frac{1}{1+\alpha_i} \text{ and } 1-\mu_i = \frac{\alpha_i}{1+\alpha_i}$$

Comment # 2

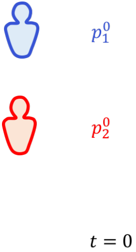
Observing Deterministic or Probabilistic Choice?



$$p_1(x,\{x,y\}) = p_2(x,\{x,y\})\frac{w_1(x)+\alpha_1}{w_1(x)+w_1(y)+\alpha_1} + p_2(y,\{x,y\})\frac{w_1(x)}{w_1(x)+w_1(y)+\alpha_1}$$

$$p_1(x,\{x,y\}) = \frac{w_1(x)+\alpha_1 p_2(x,\{x,y\})}{w_1(x)+w_1(y)+\alpha_1}$$

Dynamic Adjustment



Dynamic Adjustment

p_1^0 $\qquad$ $p_1^1 = f(w_1, p_2^0)$

p_2^0 $\qquad$ $p_2^1 = f(w_2, p_1^0)$

$t = 0$ $\qquad$ $t = 1$

Dynamic Adjustment

p_1^0 → $p_1^1 = f(w_1, p_2^0)$ → $p_1^2 = f(w_1, p_2^1)$

p_2^0 $\quad$ $p_2^1 = f(w_2, p_1^0)$ $\quad$ $p_2^2 = f(w_2, p_1^1)$

$t = 0$ $\quad$ $t = 1$ $\quad$ $t = 2$

Dynamic Adjustment

p_1^0 → $p_1^1 = f(w_1, p_2^0)$ → $p_1^2 = f(w_1, p_2^1)$ … $p_1^\infty = p_1$

p_2^0 → $p_2^1 = f(w_2, p_1^0)$ → $p_2^2 = f(w_2, p_1^1)$ … $p_2^\infty = p_2$

$t = 0$ $t = 1$ $t = 2$ $t = \infty$

Story behind our formulation

$$p_1(x, S) = \frac{w_1(x) + \alpha_1 p_2(x, S)}{\sum_{y \in S} [w_1(y) + \alpha_1 p_2(y, S)]}$$

1. Random utility with social interactions
2. Quantal response equilibrium
3. Naive learning

Story 1: Random Utility

- *Linear social interaction models*: Manski (1993), Blume et al. (2011), Jackson (2011), Blume et al. (2015)
  - $U_i(x)$ = individual private utility + social utility
  - Social utility depends on the expected behaviors of one's peers.
- *Discrete choice models with social interactions*: Blume (1993), Brock and Durlauf (2001, 2003)
  - Constant strategic complementarity
  - Rational expectations
  - Errors follow a relevant extreme value distribution

Story 1: Random Utility

- $V_i(x,S) = w_i(x) + \alpha_i p_j(x,S)$
- $U_i(x,S) = V_i(x,S)\varepsilon_i(x)$
- i.i.d. errors with a Log-logistic distribution, $f(\log \varepsilon_i) = e^{-\log \varepsilon_i} e^{-e^{-\log \varepsilon_i}}$

$$
\begin{aligned}
p_i(x,S) &= Prob\left(\log U_i(x,S) > \log U_i(y,S) \;\; \forall y \neq x\right) \\
&= Prob\left(\log \varepsilon_i(y) < \log\left(\frac{V_i(x,S)\varepsilon_i(x)}{V_i(y,S)}\right), \;\; \forall y \neq x\right) \\
& \ldots \\
&= \frac{w_i(x) + \alpha_i p_j(x,S)}{\sum_{y \in S} \left(w_i(y) + \alpha_i p_j(y,S)\right)}
\end{aligned}
$$

Story 2: Quantal response equilibrium

- A normal form game with two players Dan and Bob,
- The pay-off matrix

Bob

		x	y
Dan	x	$(w_1(x)+\alpha_1, w_2(x)+\alpha_2)$	$(w_1(x), w_2(y))$
	y	$(w_1(y), w_2(x))$	$(w_1(y)+\alpha_1, w_2(y)+\alpha_2)$

Story 2: Quantal response equilibrium

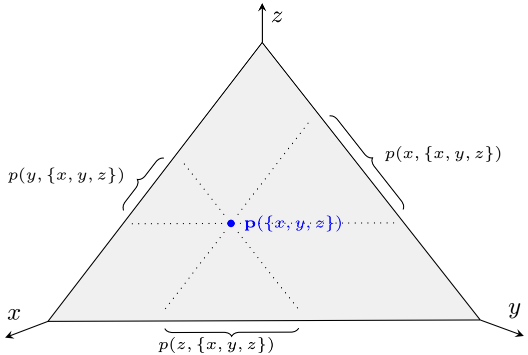
		Bob x	Bob y
Dan	x	$(w_1(x) + \alpha_1, w_2(x) + \alpha_2)$	$(w_1(x), w_2(y))$
Dan	y	$(w_1(y), w_2(x))$	$(w_1(y) + \alpha_1, w_2(y) + \alpha_2)$

- s_i is a pure strategy, σ_i is a mixed strategy for player i.
- Player i's expected payoff from s when j plays σ_j

  $u_i(s, \sigma_j) = \sigma_j(s)(w_i(s) + \alpha_i) + (1 - \sigma_j(s))w_i(s) = w_i(s) + \alpha_i\sigma_j(s).$
- Under the assumption that $U_i(s, \sigma) = u_i(s, \sigma)\varepsilon_{is}$ with i.i.d. log-logistic errors ε_{is}, the QRE outcome coincides with (p_1, p_2) of the dual interaction model.

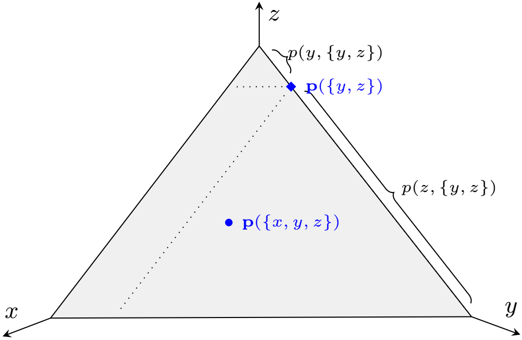
A Graphical Representation

- Consider $\mathbf{p}(\{x,y,z\}) = (p(x,\{x,y,z\}), p(y,\{x,y,z\}), p(z,\{x,y,z\}))$
- $\mathbf{p}(\{x,y,z\})$ is a point in a simplex



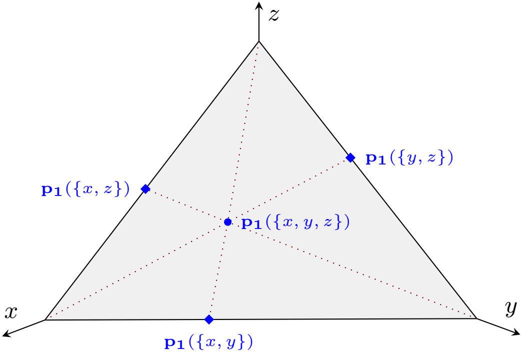
A Graphical Representation

$\mathbf{p}(\{y,z\})$ is also a point in a simplex



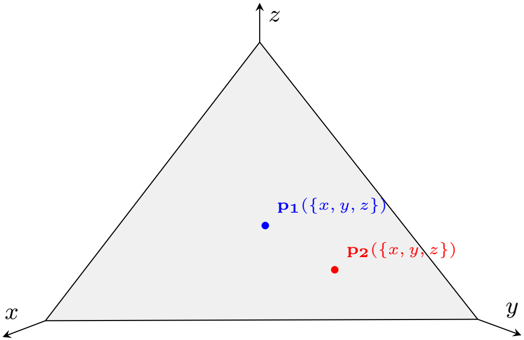
No Influence

"No Influence" $p_1(x, A) = \frac{w_1(x)}{\sum_{y \in A} w_1(y)}$

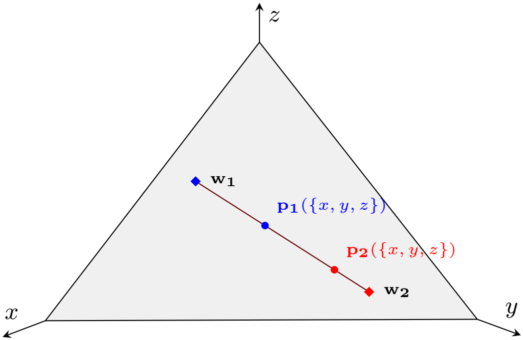


- Luce's IIA: $\frac{p_1(x,A)}{p_1(y,A)} = \frac{p_1(x,B)}{p_1(y,B)}$

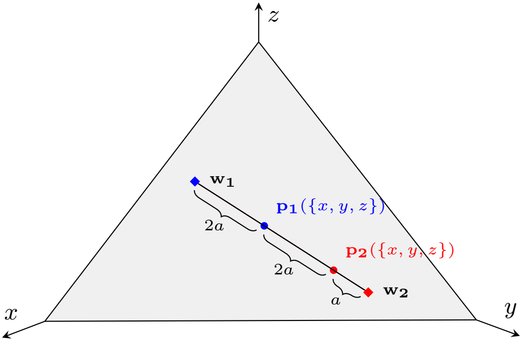
Graphical Representation



Graphical Representation



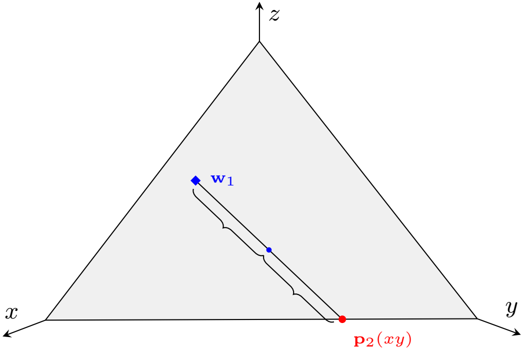
Graphical Representation



$\alpha_1 = 1$ and $\alpha_2 = .5$

Graphical Representation

What about $p_1(\{x, y\})$?



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Graphical Representation

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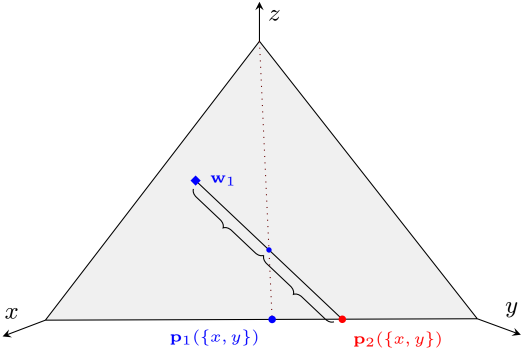
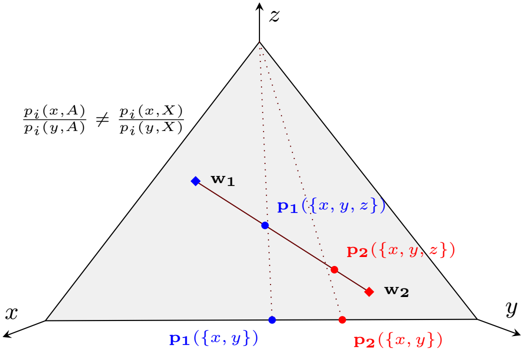


$\alpha_1 = 1$ and $\alpha_2 = .5$

Graphical Representation

Existing of Influence ⇒ IIA fails

$$\frac{p_i(x,A)}{p_i(y,A)} \neq \frac{p_i(x,X)}{p_i(y,X)}$$



Identification

- Assume the model is correct
- How can we identify parameters of the model (w_i, α_i)?
- Take two sets X and S (Minimal Data)
- Observe that $\frac{0.71}{0.29} \approx 2.5 \neq 2.3 \approx \frac{0.60}{0.26}$
- Key: Luce's IIA violation

Identification

First assume no influence and consider

$$p_i(x,S) = \frac{w_i(x)}{w_i(S)} \text{ and } p_i(x,X) = w_i(x)$$

$$\begin{aligned} d_i(x,S) =& p_i(x,S) - p_i(x,X) \\ =& p_i(x,S) + w_i(S)p_i(x,S) \\ =& (1 - w_i(S))p_i(x,S) > 0 \end{aligned}$$

Identification

In our model,

$$d_i(x,S) = \underbrace{\frac{1-w_i(S)}{1+\alpha_i}p_i(x,S)}_{\text{individual}} + \underbrace{\frac{\alpha_i}{1+\alpha_i}d_j(x,S)}_{\text{social influence}}$$

Identification

$$\frac{d_i(x,S)}{p_i(x,S)} - \frac{d_i(y,S)}{p_i(y,S)} = \frac{\alpha_i}{1+\alpha_i}\left[\frac{d_j(x,S)}{p_i(x,S)} - \frac{d_j(y,S)}{p_i(y,S)}\right]$$

Identification

$$\frac{\alpha_i}{1+\alpha_i} = \frac{\frac{d_i(x,S)}{p_i(x,S)} - \frac{d_i(y,S)}{p_i(y,S)}}{\frac{d_j(x,S)}{p_i(x,S)} - \frac{d_j(y,S)}{p_i(y,S)}}$$

- What about w_i?

$$w_i(x) = p_i(x,X) + \alpha_i(p_i(x,X) - p_j(x,X))$$

Revisit Example

	Dan	Bob		Dan	Bob
walk outside	0.71	0.78		0.60	0.70
exercise home	0.29	0.22		0.26	0.19
go to the gym				0.14	0.11

$$\frac{\alpha_1}{1+\alpha_1} = \frac{\frac{d_i(w,S)}{p_i(w,S)} - \frac{d_i(e,S)}{p_i(e,S)}}{\frac{d_j(w,S)}{p_i(w,S)} - \frac{d_j(e,S)}{p_i(e,S)}} = \frac{\frac{0.11}{0.71} - \frac{0.03}{0.29}}{\frac{0.08}{0.71} - \frac{0.03}{0.29}} = \frac{5}{6}$$

- $\alpha_1 : 5$ and $\alpha_2 : 1$
- $w_1 : 0.1, 0.6, 0.3$ and $w_2 : 0.8, 0.12, 0.08$

Identification

- Quantify Influence and Identify Preference
- Minimal Data
- Can we falsify this model?

Characterization

Define $\beta_i(x,y,S)$ for all distinct $x,y \in S \neq X$ with $\dfrac{d_j(x,S)}{p_i(x,S)} - \dfrac{d_j(y,S)}{p_i(y,S)} \neq 0$ as follows:

$$\frac{d_i(x,S)}{p_i(x,S)} - \frac{d_i(y,S)}{p_i(y,S)} = \beta_i(x,y,S)\left[\frac{d_j(x,S)}{p_i(x,S)} - \frac{d_j(y,S)}{p_i(y,S)}\right] \tag{1}$$

Independence [*I*].

i) $\beta_i(x,y,S)(:= \beta_i)$ is independent of S, x, y, and

ii) β_i satisfies (1) for all $S \neq X$ and distinct $x, y \in S$.

Characterization

Positive Uniform Boundedness: $\beta_i(x, y, S) < \min_{z \in X} \left\{ \frac{p_i(z,X)}{p_j(z,X)} \right\}$, for all S and $x, y \in S$.

Non-negativeness: $\beta_i(x, y, S) \geq 0$, for all S and $x, y \in S$.

Characterization

Theorem

*Suppose p_i does not satisfy IIA at least for one individual. Then (p_1, p_2) has a **dual interaction** representation with $\alpha_1, \alpha_2 \in \mathbb{R}_+$ if and only if Axiom 1-3 hold. Moreover, $(w_1, w_2, \alpha_1, \alpha_2)$ is uniquely identified.*

Summary

- Our aim was
  - propose a simple and intuitive model
  - detect interaction from observed choice behavior
  - quantify influence and identify preference
  - minimal data requirement (one menu variation)

Generalization

$$p_i(x,S) = \frac{U_i(x|S,\alpha_i,p_j)}{\sum_{y\in S} U_i(y|S,\alpha_i,p_j)}$$

- The current paper: $U_i(x|S,\alpha_i,p_j) = w_i(x) + \alpha_i p_j(x,S)$
- $U_i^*(x|S,\alpha_i,p_j) = (1-\alpha_i)\frac{w_i(x)}{w_i(S)} + \alpha_i p_j(x,S)$
- Many more...

Uniqueness and Stability

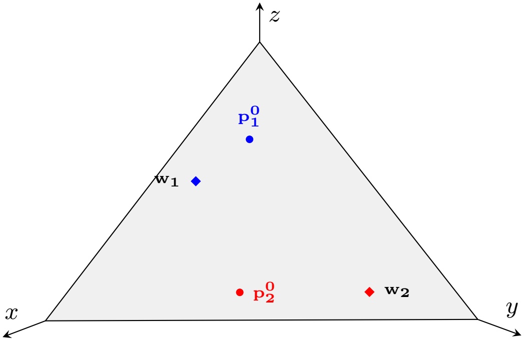
- Uniqueness of "equilibrium"
- Stability of the "equilibrium"

Uniqueness and Stability

- Uniqueness of "equilibrium":
  - For any $(w_1, w_2, \alpha_1, \alpha_2)$, is there a unique pair of (p_1^*, p_2^*) consistent with the model?

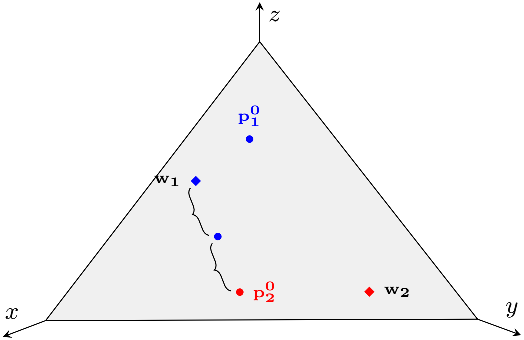
- Stability of the equilibrium:
  - Let (p_1^0, p_2^0) be the initial behavior
  - Assume the dual interaction model
  - What happens in the long run?

Proof by Picture



$\alpha_1 = 1$ and $\alpha_2 = .5$

Proof by Picture



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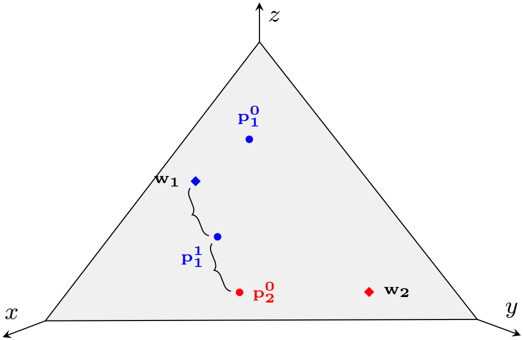

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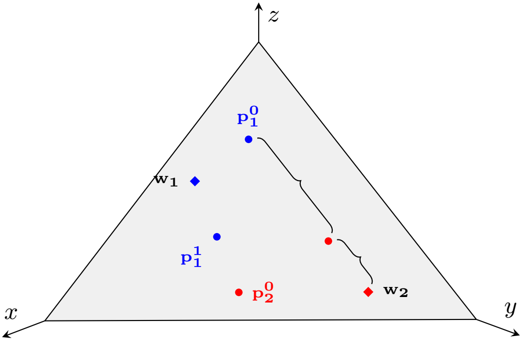

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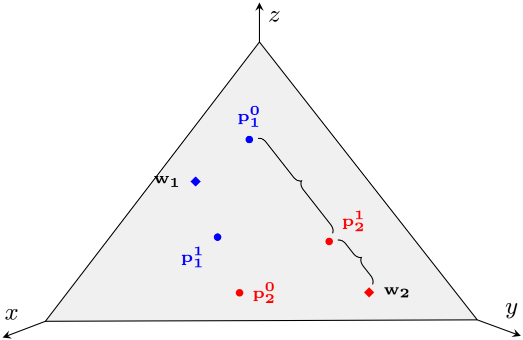

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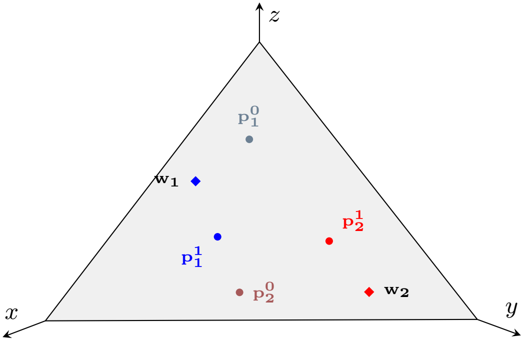

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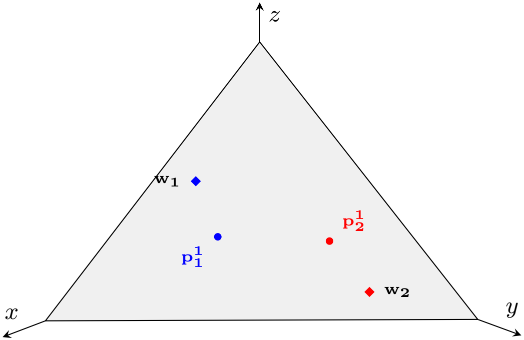

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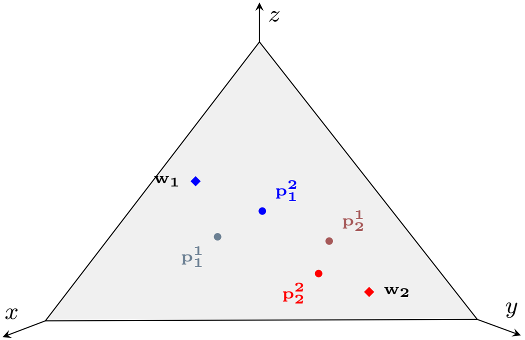

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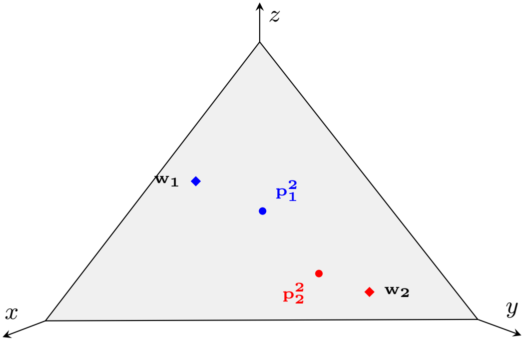

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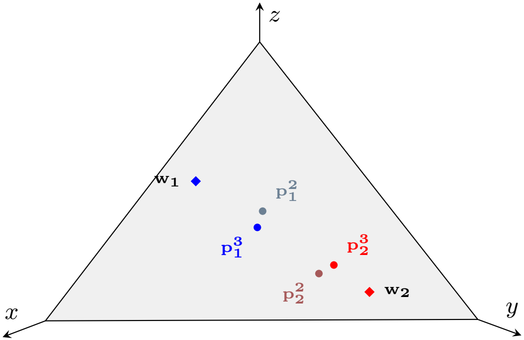
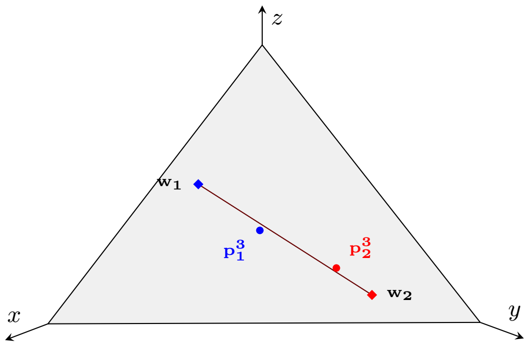
Proof by Picture



$\alpha_1 = 1$ and $\alpha_2 = .5$

Proof by Picture



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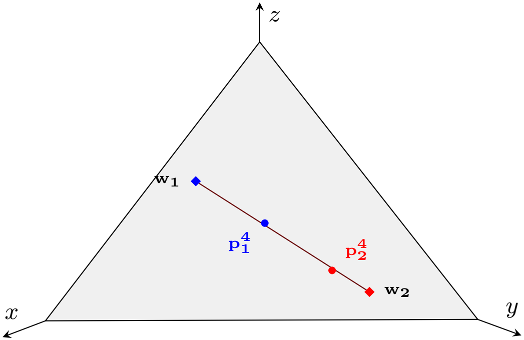

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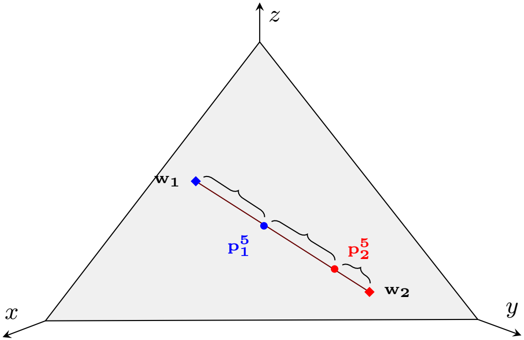

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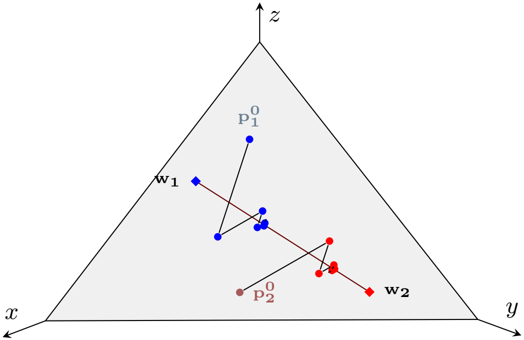


$\alpha_1 = 1$ and $\alpha_2 = .5$

Uniqueness and Stability

Theorem

Let $w_i \gg 0$ *and* $\alpha_i \geq 0$ *for each* $i \in \{1,2\}$. *Let* $S \in 2^X \setminus \{\varnothing\}$. *Then there are unique* $p_i^*(S) \in \Delta_{++}(S)$ *for which for all* $x \in S$,

$$p_i^*(x,S) = \frac{w_i(x) + \alpha_i p_j^*(x,S)}{\sum_{y \in S} w_i(y) + \alpha_i p_j^*(y,S)}.$$

Further, let $(p_1^0, p_2^0) \in \Delta(S) \times \Delta(S)$. *Define for each* $i \in \{1,2\}$ *and* $t \geq 1$, $p_i^t(\cdot, S) \in \Delta(S)$ *via*

$$p_i^t(x,S) \equiv \frac{w_i(x) + \alpha_i p_j^{t-1}(x,S)}{\sum_{y \in S} w_i(y) + \alpha_i p_j^{t-1}(y,S)}.$$

Then for each $i \in \{1,2\}$, $\lim_{t \to \infty} p_i^t = p_i^*$.

Dynamic Identification

- What about identification in this dynamic setting? Any inference if we were to observe $...p_1^{t-1}, p_1^t...$?
- Yes! Although the behavior changes every period, it changes consistently. Same identification strategy:

$$\beta_i(x,y,S) = \frac{\dfrac{d_i^t(x,S)}{p_i^t(x,S)} - \dfrac{d_i^t(y,S)}{p_i^t(y,S)}}{\dfrac{d_j^{t-1}(x,S)}{p_i^t(x,S)} - \dfrac{d_j^{t-1}(y,S)}{p_i^t(y,S)}} = \frac{\alpha_i}{1+\alpha_i}$$

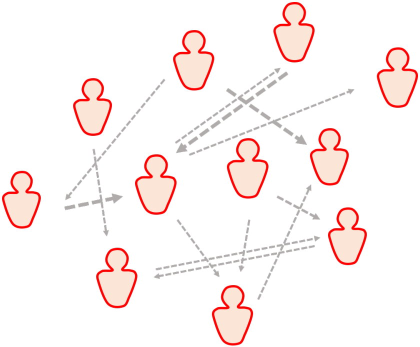
.

$$w_i(x) = p_i^t(x,X) + \alpha_i(p_i^t(x,X) - p_j^{t-1}(x,X))$$

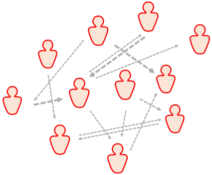
Extensions

- Multi-agent Interaction
- Negative Interaction

Multi-agent Interaction



Multi-agent Interaction



Let N finite set of agents with $(p_1, p_2, ..., p_n)$.

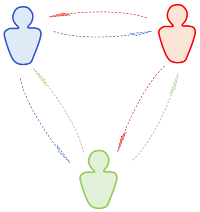
Definition

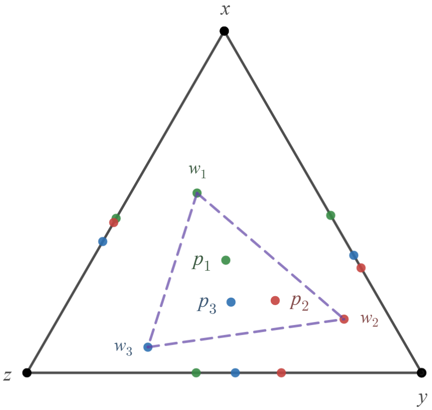
$(p_1, p_2, ..., p_n)$ has a **social interaction** representation if for each $i \in N$ there exist $w_i : X \to (0,1)$ with $\sum_{x \in X} w_i(x) = 1$ and $\boldsymbol{\alpha}_i \in \mathbb{R}^{n-1}$ such that

$$p_i(x, S) = \frac{w_i(x) + \boldsymbol{\alpha}_i \cdot \mathbf{p}_{-i}(x, S)}{\sum_{y \in S} [w_i(y) + \boldsymbol{\alpha}_i \cdot \mathbf{p}_{-i}(y, S)]}$$

for all $x \in S$ and for all S.

Multi-agent Interaction





Characterization

$$\boldsymbol{\gamma}_i \cdot \left(\frac{\mathbf{d}_{-i}(x,S)}{p_i(x,S)} - \frac{\mathbf{d}_{-i}(y,S)}{p_i(y,S)} \right) = \frac{d_i(x,S)}{p_i(x,S)} - \frac{d_i(y,S)}{p_i(y,S)}. \qquad (2)$$

$$\mathcal{B}_i = \{\boldsymbol{\gamma}_i \in R^{n-1} \, | \boldsymbol{\gamma}_i \text{ solves (2) for any } S \text{ and distinct } x, y \in S\}$$

N-Independence [*N-I*]. *$\mathcal{B}_i$ is nonempty.*

Characterization

N-Independence [*N-I*]. *$\mathcal{B}_i$ is nonempty.*

N-Uniform Boundedness. [*N-UB*] *For all $z \in X$, $p_i(z, X) > \boldsymbol{\gamma}_i \cdot \boldsymbol{p}_{-i}(z, X)$ for some $\boldsymbol{\gamma}_i \in \mathcal{B}_i$ with $\boldsymbol{\gamma}_i \in R_+^{n-1}$.*

Characterization

Theorem

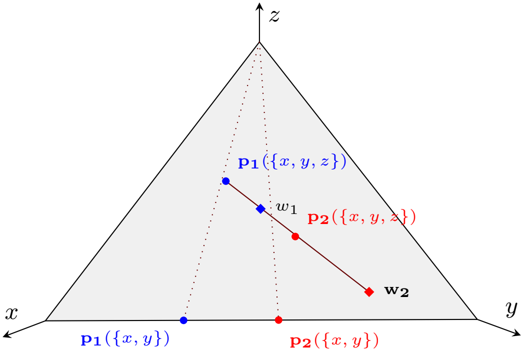
Let distinct p_i. Then $(p_1, p_2, ..., p_n)$ has a ***social interaction*** *representation if and only if n-independence, n-uniform boundedness, and n-nonnegativeness hold. Moreover, $\{w_i, \boldsymbol{\alpha}_i \geq 0\}_{i \in N}$ are uniquely identified.*

Negative Interactions

- Fashions and fads
- The choice of a fashion product not only signals which social group you would like to identify with but also signals who you would like to differentiate from (Pesendorfer, '95)
- Among criminals competition for resources governs the need for negative interactions (Glaeser et al, '96)
- Lots of evidence but less theoretical work

Negative Interactions

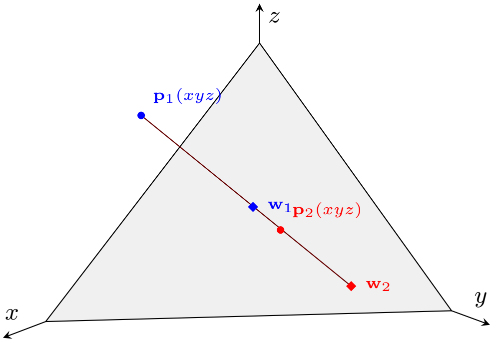
How to incorporate negative influence: let $\alpha_i \in R$



$\alpha_1 = -.5$ and $\alpha_2 = 1$

Negative Interactions

- **Existence of representation:** Not every combination of $(w_1, w_2, \alpha_1, \alpha_2)$ yield a dual interaction representation



Negative Interactions: Characterization

Fairly straightforward:

Let $i \neq j$. For any $S \neq X$, and any $x, y \in S$ for which $x \neq y$, define

$$\gamma_i(x,y,S) \equiv \frac{1}{\beta_i(x,y,S)} = \frac{\frac{d_j(x,S)}{p_i(x,S)} - \frac{d_j(y,S)}{p_i(y,S)}}{\frac{d_i(x,S)}{p_i(x,S)} - \frac{d_i(y,S)}{p_i(y,S)}}.$$

Conditional Independence: If p_i does not have a Luce representation, then $\gamma_i(x,y,S)$ is independent of S, x, and y.

Uniform Boundedness: For all $S \neq X$ and $x, y \in S$

$$\gamma_i(x,y,S) \notin \left[\min_{z \in X}\left\{\frac{p_j(z,X)}{p_i(z,X)}\right\}, \max_{z \in X}\left\{\frac{p_j(z,X)}{p_i(z,X)}\right\}\right].$$

Negative Interactions: Characterization

Theorem

Let $p_1 \neq p_2$. (p_1, p_2) has a ***dual interaction*** *representation with $\alpha_1, \alpha_2 \in \mathbb{R}$ if and only if it satisfies conditional independence and uniform boundedness. Moreover, $(w_1, w_2, \alpha_1, \alpha_2)$ is uniquely identified.*