Reputation Concerns in Risky Experimentation

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September 6, 2020

Motivation

- Reputation concerns are important in many facets of our life.
- Even more so in the context of exploratory activities that are typically undertaken by professionals and experts, such as entrepreneurs, politicians and scientists.
- Reputation is an indispensable asset for advancing their (or our) careers.

Motivation

- Reputation in risky experimentation is established by both successes and failures, but in ways that are not as straightforward as they seem.
- Consider, for instance, the decision to abandon a risky venture.
  - Does a better entrepreneur persist longer, because he is better able to implement a good idea?
  - Or, do he quit earlier, because he is quicker to recognize its futility?
- In the context of experimentation, even success is not an unambiguous sign of competence; success that arrives very late may be taken as a sign of mediocrity.

Motivation

- More broadly, high-ability agents are more likely to achieve success for a given project quality, but also learn faster that their project is not promising.
- Crude intuition suggests that the way reputation concerns work in the context of risky experimentation is qualitatively different from that in other standard economic contexts characterized by *single-crossing preferences.*
- But, precisely in what ways?

Overview of our results

- We identify two effects that relevant for signaling in this context.
  - *Ability effect*: the high type persists longer because he is better able to implement a project.
  - *Learning effect:* the high type quits earlier because his posterior belief declines more quickly.
- The key to our analysis is not which effect outweighs the other.
- Rather, the ability effect dominates in early stages of experimentation while the learning effect dominates in later stages.
- The interaction of these effects gives rise to *double-crossing preferences* for signaling.

Overview of our results

- The double-crossing property tends to generate pooling (under D1 refinement).
  - It places an endogenous constraint on how late (or early) the high type can quit.
  - But if the high type cannot quit too late, it is easier for the low type to mimic the high type.
  - Clear contrast to a standard setting which predicts the least-cost separating equilibrium (or "Riley outcome").
- Dynamics matters, especially when the reward to success depends on reputation.

Literature

- Strategic experimentation: Bolton and Harris (1999); Keller et al. (2005)
- Reputation models of experimentation: Bobtcheff and Levy (2017); Bonatti and Horner (2017); Thomas (2019); Halac and Kremer (2020)
- Dynamic signaling: Bar-Isaac (2003); Daley and Green (2012); Gul and Pesendorfer (2012); Lee and Liu (2013)

Model: setup

- An agent undertakes a risky project with unknown quality and at the same time signals his competence to the market.
- If the quality is bad, it will never generate success no matter how much time the agent spends working on it (e.g., working on a false conjecture).
- If the quality is good, it will generate success at some random time τ (if the agent has not abandoned the project by that time).
- There is a flow cost of working on the project: $c\mathrm{d}t$.

Model: setup

- The agent differs in ability, either *high* (H) or *low* (L).
- Let $f_i(\tau)$ be the density function of τ for $i = H.L$ with $F_i(\tau)$ the corresponding distribution.
  - the (conditional) hazard rate $f_i/(1 - F_i)$ is weakly decreasing.
  - Monotone likelihood ratio: f_H/f_L is strictly decreasing.
- Remark: while most existing works in the literature assume "exponential bandits," we here allow for more general form, only assuming that the hazard rate is decreasing, with exponential bandits as a special case.

Model: setup

- The state of nature is two-dimensional, defined over project quality (good or bad) and ability (high or low).
- The ability type is the agent's *private information* (so that there is signaling).
  - The market's prior belief that the agent is high is q_0.
- The project quality is not known to anyone initially, and needs to be uncovered via experimentation.
  - The common prior belief that the project is good is p_0.
  - We will later allow for the possibility that p_0 is type-dependent.

Model: setup

- Time is continuous and extends from zero to infinity.
- At each instant, the agent decides whether to continue working the project or to abandon it.
- Without loss of generality, we assume that abandoning the project is irreversible.
- The game ends either when the agent achieves success or abandons the project (which we often refer to as "failure").

Model: payoffs

- There is a competitive labor market which pays the agent's expected productivity.
- W_i is the productivity of type i after success; w_i after failure.
- Let q_t denote the agent's reputation (the market's belief).
- The agent receives $W(\tau) = q_\tau W_H + (1 - q_\tau) W_L$ if he succeeds at τ and $w(t) = q_t w_H + (1 - q_t) w_L$ if he abandons the project at t.
- Regard $W(\tau)$ and $w(t)$ as the continuation payoffs of success and failure, respectively.

Model: payoffs

- Assume $w_H > w_L$ and $W_H \geq W_L$.
- The difference between $W_H = W_L$ and $W_H > W_L$ will be crucial, so we will analyze them separately.
  - *Exit signaling:* $W_H = W_L$, and only the exit payoffs depends on reputation.
  - *Breakthrough signaling:* $W_H > W_L$, so that the reward to success also depends on reputation.
- Also, $W_L > w_H$ (which can be relaxed but for simplicity).

Model: interpretation

- We take venture startups as a leading example of our model.
  - Success is uncertain and its timing is stochastic in any business startup.
  - Reputation is a crucial input for developing a new business (in attracting financial capital and talents to work with).
- The difference between $W_H = W_L$ and $W_H > W_L$ captures where in the experimentation process the agent stands.
  - Successfully developing a prototype product is only a step toward a bigger goal $\Rightarrow$ reputation after success is still important (W_H larger than W_L).
  - Gaining global market recognition is valuable in and of itself $\Rightarrow$ reputation after success is less relevant (W_H close to W_L).
- Other examples: academia; politics ...

Model: equilibrium selection

- We adopt perfect Baysian equilibrium as the solution concept.
- As typical in signaling models, this does not pin down a unique equilibrium.
- We adopt the Cho-Kreps D1 criterion for refinement.
- D1 is a particularly useful concept in our framework as it predicts a unique outcome (whereas the Intuitive Criterion does not).

Exit signaling

- The updated belief about the agent's type at the time of termination depends on: (1) inference based on t and the strategies of the two types; (2) observation about τ.
- The *interim belief* is defined as

$$\hat{q} = \Pr[\text{high type} \mid \sigma_L, \sigma_H, \text{stops at t}].$$

- The belief based on both (1) and (2) are then given by

$$q = \Pr[\text{high type} \mid \sigma_L, \sigma_H, \text{stops at t}, \tau > t].$$

- By Bayes' rule,

$$q_t = r(t; \hat{q}) := \frac{\hat{q}(1 - p_0 F_H(t))}{\hat{q}(1 - p_0 F_H(t)) + (1 - \hat{q})(1 - p_0 F_L(t))}.$$

Exit signaling

- The (unconditional) hazard rate is

$$g_i(t) = \frac{p_0 f_i(t)}{1 - p_0 F_i(t)} = \left(\frac{f_i(t)}{1 - F_i(t)}\right)\left(\frac{p_0(1 - F_i(t))}{p_0(1 - F_i(t)) + 1 - p_0}\right).$$

- The first term is the hazard rate (condition on good quality), which is always *higher* for the high type due to MLR property.
- The second terms is the posterior that the project is good, conditional on no success having occurred by t, which is always *lower* for the high type.
- The former captures the ability effect while the latter captures the learning effect.

Exit signaling

- The following result is crucial for analyzing signaling in risky experimentation.

Lemma

The hazard rate g_i is strictly decreasing for $i = H, L$. There exists a unique $\hat{t}$ such that $g_H(t) > g_L(t)$ iff $t < \hat{t}$ and $g_H(t) < g_L(t)$ iff $t > \hat{t}$.

Double-crossing preferences

- The objective function when $W_H = W_L = W$ reduces to

$$U_i(s,\hat{q}) = \int_0^s e^{-\rho\tau} p_0 f_i(\tau)\left[W - C(\tau)\right] \mathrm{d}\tau$$
$$+ e^{-\rho s}(1 - p_0 F_i(s))\left[-C(s) + w_L + r(s;\hat{q})(w_H - w_L)\right].$$

- The marginal rate of substitution between stopping time s and interim belief $\hat{q}$, denoted $MRS_i(s,\hat{q})$, is

$$\frac{g_i(s)[W - w_L - r(s;\hat{q})(w_H - w_L)] - \rho(w_L + r(s;\hat{q})(w_H - w_L)) - c + (\partial r/\partial s)(w_H - w_L)}{(\partial r/\partial \hat{q})(w_H - w_L)}.$$

- Observe that MRS depends on type only through g_i.

Double-crossing preferences

- $MRS_H(s, \hat{q}) > MRS_L(s, \hat{q})$ for $s < \hat{t}$ and $MRS_H(s, \hat{q}) < MRS_L(s, \hat{q})$ for $s > \hat{t}$.
- Since $g_H - g_L$ is single-crossing from above, $MRS_H - MRS_L$ is also single-crossing.
- This means that indifference curves cross twice ("double-crossing") with the indifference curve of the high type (ICH) more "convex" than that of the low type (ICL)
- Signaling incentives are totally different before and after $\hat{t}$.
  - Before $\hat{t}$, ICH is less steep than ICL (as in standard signaling model).
  - After $\hat{t}$, this relationship flips, with ICH becoming steeper than ICL.
- This feature yields crucial implications for off-path inferences under D1.

D1 under double-crossing preferences

- D1 is one of the most commonly used refinement concepts, perhaps along with Intuitive Criterion, in signaling models.
- D1 assigns probability 0 to type θ after a deviation if there is another type θ' who would benefit more from the deviation.
- According to D1, type θ' benefits more if the set of responses (or posterior beliefs) that make θ' willing to deviate is strictly larger than the set of responses that make type θ willing to deviate.
- D1 is generally stronger than IC.

D1 under double-crossing preferences

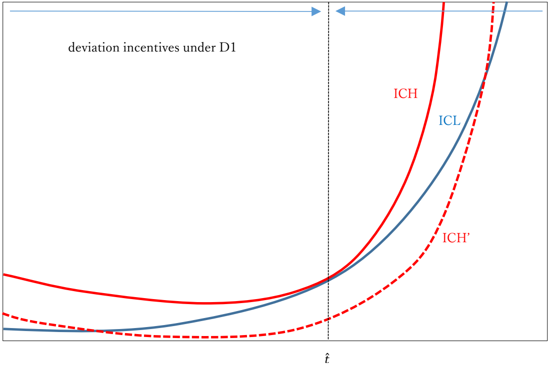

Figure: For $s < \hat{t}$, ICH crosses ICL from above, giving an upward deviation incentive; for $s > \hat{t}$, the opposite is true, giving a downward deviation incentive. Deviation incentives point towards $\hat{t}$.

Equilibrium with exit signaling

- Let s_i^* denote the full-information optimal stopping time for type i, which solves

$$g_i(s_i^*)(W - w_i) - (\rho w_i + c) = 0.$$

- In any signaling model, the low type cannot do worse than choosing s_L^* and revealing his true type.
- It is useful to define $\underline{s}$ and $\overline{s}$ such that

$$U_L(\underline{s}, 1) = U_L(s_L^*, 0) = U_L(\overline{s}, 1).$$

- Efficient separation obtains if $s_H \notin (\underline{s}, \overline{s})$; we will focus on the case $s_H \in (\underline{s}, \overline{s})$.

Separating equilibrium

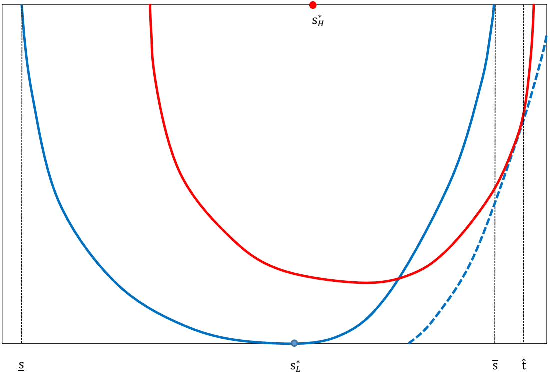

Figure: $\hat{t}$ lies outside of $(\underline{s}, \overline{s})$. In this case, the high type stopping at $\overline{s}$ and the low type stopping at s_L^* is the only equilibrium that survives D1.

Pooling equilibrium

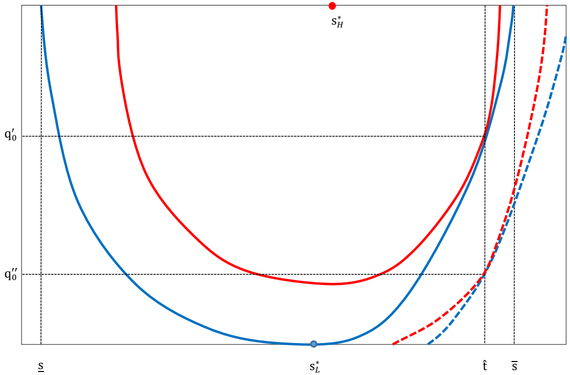

Figure: $\hat{t}$ lies inside $(\underline{s}, \overline{s})$. In this case, there will be some pooling under D1. If the prior is larger than q_0', there will be full pooling; if not, there will be partial pooling.

Equilibrium characterization

- Equilibrium is always unique under D1.
- When $\hat{t} \in (\underline{s}, \overline{s})$, some form of pooling emerges.
- The low type "holds out" for the prospect of pooling with the high type, and the high type conforms to avoid adverse inference.
- Clear contrast to a standard model with single-crossing preferences where the low type never selects an action higher than the full-information optimal level and the high type can always go far enough to separate.

Equilibrium characterization

- A little technical remark is that D1 and Intuitive Criterion make slightly different predictions, even though we have only two types.
- Since IC is generally weaker than D1, all equilibria that survive D1 also survive IC.
- However, IC cannot rule out the least-cost separating equilibrium when there is a D1 pooling equilibrium.
- We argue that D1 is the more reasonable criterion as it narrows the set of equilibria down to a unique one.

Breakthrough signaling

- Now let $W_H > W_L$, so that the reward to success is dependent on reputation.
- The case of "breakthrough signaling" is much more complicated, because the timing of success is stochastic.
- In particular, we need to keep track of two interim beliefs, $\hat{q}$ (the interim belief at the time of termination) and $\tilde{q}$ (at the time of success).
- We can no longer draw a nice two-dimensional picture as in the case of exit signaling.

Breakthrough signaling

- The *interim belief* in case of success is defined as

$$\tilde{q} = \Pr[\text{high type} \mid \sigma_L, \sigma_H, \text{has not stopped by t}],$$

  which is (most likely) different from $\hat{q}$.
- By Bayes' rule, the market's belief of the agent who succeeds at t is

$$q_t = R(t; \tilde{q}) := \frac{\tilde{q} f_H(t)}{\tilde{q} f_H(t) + (1 - \tilde{q}) f_L(t)}.$$

- The reputation upon success may be higher or lower than the reputation upon failure: success that comes too late may be a sign of mediocrity.

Breakthrough signaling

- Even with breakthrough signaling, the following property continues to hold.

Lemma

If both types of agent abandon the risky project at some t with positive probability in equilibrium, then $t = \hat{t}$.

Breakthrough signaling

- We can now define a counterpart of s_i^*: let $s_i^*(\tilde{q})$ be the solution to

$$g_i(s_i^*(\tilde{q}))[W_L + R(s_i^*(\tilde{q}); \tilde{q})(W_H - W_L) - w_i] - (\rho w_i + c) = 0.$$

- The optimal stopping rule depends on the interim belief $\tilde{q}$.
- When $W_H = W_L$, $s_L^*(0)$ equals s_L^* and $s_H^*(1)$ equals s_H^*.
- A higher interim belief for stayers raises the reward to success and delays quitting.

High type quits first

- The analysis of breakthrough signaling depends crucially on which type quits first.
- If $s_L^*(q_0) \geq \hat{t}$, which in turn implies $s_L^*(q_0) > s_H^*(q_0)$, the high type quits first.
- In equilibrium, the high type tends to quit prematurely because the reputational value of success is smaller with more low types around.

Low type quits first

- A more noticeable difference can be observed when $s_L^*(q_0) < \hat{t}$, which in turn implies $s_H^*(q_0) < \hat{t}$, in which case the low type quits first.
- In equilibrium, no type quits until $s_L^*(q_0)$.
- At $s_L^*(q_0)$, a low type agent is ready to quit.
- If a low type agent quits, however, the interim belief rises and so does the reputational value of success.

Low type quits first

- It is in effect a game of strategic substitutes, or a war of attrition, among low type agents.
- Each low type agent waits for others to drop out.
- In equilibrium, the low type is indifferent over some interval (up to some point).
- As time draws closer to the equilibrium stopping time of the high type, the low type may stop quitting and hold out.
- In this case, equilibrium entails continuous randomization, followed by a hold-out phase which leads to a mass exit.

The role of reputation concerns

- The extent of reputation concerns can be measured by $w_H - w_L$ and $W_H - W_L$.
- Strong reputation concerns when they are large.
- Our model provides clear predictions regarding the extent of reputation concerns.

The role of reputation concerns

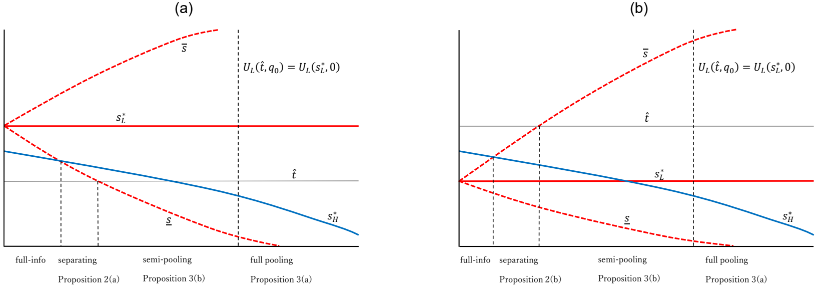

Figure: Equilibrium as a function of w_H for a given w_L with $W_H = W_L$. Panel (a) depicts the case of $s_L^* > \hat{t}$. Panel (b) depicts the case of $s_L^* < \hat{t}$. In both cases, in the range where the incentive compatibility is binding ($s_H^* \in (\underline{s}, \overline{s})$), the equilibrium changes from separating to semi-pooling and to full pooling as w_H increases.

The role of reputation concerns

- Greater concerns for reputation as measured by $w_H - w_L$ induce *homogenization of quitting times* between types.
- The key object is $(\underline{s}, \overline{s})$, which is the interval in which the low type is willing to mimic the high type.
- An increase in w_H for a given w_L widens $(\underline{s}, \overline{s})$ and contains $\hat{t}$ at some point.
- When $\hat{t} \in (\underline{w}, \overline{w})$, some form of pooling emerges.

The role of reputation concerns

- The type of equilibrium is largely determined by $w_H - w_L$.
- An increase in W_H for a given W_L still induces pooling.
- When the low type quits first, a larger $W_H - W_L$ results in more incentive to hold out, which slows down the dynamic separation of types.

The role of prior belief

- In standard signaling models, D1 always selects the least-cost separating equilibrium which is belief-free.
- This is somewhat disturbing, as we cannot discuss the role of reputation.
- In our model, a higher prior leads to more pooling.
- With breakthrough signaling, the welfare effect of an increase in q_0 is ambiguous: it is positive when the high type quits first and negative when the low type quits first.

Implementation versus identification

- Suppose the project quality is type-dependent.
- Let p_0^i denote the prior probability that the project is good, where $1 > p_0^H > p_0^L > 0$; i.e., the high type is better at discovering ideas or identifying promising projects.
- Consider an exponential bandit model where $f_i(\tau) = \lambda_i e^{-\lambda_i \tau}$.
- The double-crossing property still holds and we can follow the same procedure to characterize equilibria.

Implementation versus identification

- High-type and low-type agents are different along two dimensions: the ability to implement a project (λ_i) and the ability to identify a good project (p_0^i).
- Which one is more important depends on the context.
  - If the agent has discretion over what to do (delegation), p_0^i should depend more on agent type.
  - If the agent simply works on the project assigned to him (centralization), the prior should not differ much between the types.
- For fixed p_0^H and p_0^L, an increase in $\lambda_H - \lambda_L$ makes implementation ability relatively more important than identification ability.

Implementation versus identification

- For a given λ_L, define $\hat{t}$ as a function of λ_H.
- We can show that $\hat{t}(\lambda_H)$ is strictly decreasing in λ_H.
- Pooling is the result when λ_H is in some intermediate range.
- If $w_H - w_L$ is sufficiently large, there is a threshold such that the equilibrium is pooling iff λ_H is above the threshold.

Implementation versus identification

- The result offers some implications for delegation.
- Consider a principal who decides whether to delegate the authority to terminate a project at the outset.
- If the principal retains the authority (centralization), she can stop at her optimal timing, but without any information about the agent.
- Centralization is unambiguously the better choice when a polling equilibrium is expected.
- Centralization is more valuable in environments implementation ability is more tested.

Model with a continuum of types

- In a companion paper (Chen et al, 2020), we consider a general model of signaling under double-crossing preferences which include the current model as a special case.
- We find that equilibrium under double-crossing preferences exhibits a particular form of signaling.
- There is a threshold type below which they are fully separated and above which they are clustered in some way.

Model with a continuum of types

- This has a precise counterpart in our model: equilibrium in our model is characterized by the low type's indifference condition.
- With a continuum of types, equilibrium is characterized by the threshold type's indifference condition.
- Our main predictions carry over to settings with more types.

An application to venture startups

- There are many potential remedies for signaling distortions, with centralization being one of them.
- The principal can certainly do better, if she can commit to and enforce a more elaborated scheme.
- We discuss two remedies that are particular relevant for venture startups.
  - *Valley of death:* Induce low type agents to quit early when they tend to persist.
  - *Startup subsidies:* Induce high type agents to persist longer.

Conclusion

- A general model is developed which encompasses a broad class of learning processes and model specifications.
- A complete characterization of D1 equilibria is obtained based on the double-crossing property.
- Our analysis illustrate how reputation concerns distort the project termination decision, often giving rise to pooling in which the high type quits prematurely and the low type holds out.
- A form of dynamic inefficiency is also identified.