Teaching Materials for "Optimal Retirement Policies with Present-Biased Agents"

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Abstract

This note serves as a companion piece to Yu (2020), and provides an intuitive explanation of how off-path allocations help relax the incentive constraints of present-biased agents using indifference curves in a two-productivity type Mirrlees setting. First, I explain how the preference arbitrage mechanism (PAM) screens productivity when all present-biased agents are fully naïve and share the same degree of present bias. Then, I explain how the conditional commitment mechanism (CCM) screend productivity when all agents are fully sophisticated and share the same degree of present bias. This note does not cover the full mechanism of Yu (2020), which combines PAM and CCM to perform multidimensional screening on sophistication, present bias, and productivity.

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1 Two-Type Case for PAM

To see how the imaginary allocations within PAM relax the incentive constraints, consider an economy with fully naïve present-biased agents who share the same present bias β. There are only two productivity types: $\Theta = \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L$. The relevant deviation is for H-agents to mimic L-agents. In particular, assume that the utility from consumption $u(c)$ is unbounded (Assumption 1 of Yu (2020)). In this setup, I will show how the government can elicit the private productivity with imaginary allocations and implement the efficient allocation.

Agents report their productivity $\theta_m \in \Theta$ and are assigned the efficient allocation for the reported productivity in $t = 0 : \left(c_0^*, y_{m,0}^*\right)$. (Recall that the efficient allocation satisfies full insurance, so $c_{H,t}^* = c_{L,t}^* = c_t^*$.) In $t = 1$, agents face a menu of allocations. In addition to the efficient allocations, PAM contains the imaginary allocation $c_{H,1}^I$ and $c_{H,2}^I$ in the menu for H-agents:

$$C_H = \left\{ \left(c_1^*, y_{H,1}^*, c_2^*\right), \left(c_{H,1}^I, y_{H,1}^*, c_{H,2}^I\right) \right\},$$

while the menu for L-agents consists only of the efficient allocation:

$$C_L = \left\{ \left(c_1^*, y_{L,1}^*, c_2^*\right) \right\}.$$

The government constructs the imaginary allocation such that consumption is backloaded, $u'\left(c_{H,1}^I\right) > u'\left(c_{H,2}^I\right)$. Since non-sophisticated H-agents in $t = 0$ over-estimate the value of retirement consumption $c_{H,2}^I$ to their futureselves, they expect to choose the backloaded imaginary consumption. However, they choose the efficient allocation, which has a higher consumption in $t = 1$ instead. From the H-agents' perspective in $t = 0$, the continuation utility from consuming the imaginary allocation $\beta\left[u\left(c_{H,1}^I\right) + u\left(c_{H,2}^I\right)\right]$ acts as a transfer, but in reality their futureselves will not choose it. The standard single-crossing in productivity implies that H-agents are incentivized to increase output in exchange for more transfers. Since utility is unbounded, the government can continually increasing $c^I_{H,2}$ until it fully relaxes the informational constraints:

$$u\left(c_0^*\right) - h\left(\frac{y^*_{H,0}}{\theta_H}\right) + \beta\left[u\left(c^I_{H,1}\right) - h\left(\frac{y^*_{H,1}}{\theta_H}\right) + u\left(c^I_{H,2}\right)\right] \\ \geq u\left(c_0^*\right) - h\left(\frac{y^*_{L,0}}{\theta_H}\right) + \beta\left[u\left(c_1^*\right) - h\left(\frac{y^*_{L,1}}{\theta_H}\right) + u\left(c_2^*\right)\right].$$

Notice that the naïve H-agent erroneously expects to choose the imaginary allocation over the efficient allocation from the menu C_H due to the preference arbitrage constraint:

$$u\left(c^I_{H,1}\right) + u\left(c^I_{H,2}\right) \geq u\left(c_1^*\right) + u\left(c_2^*\right).$$

When the government increases the imaginary retirement consumption $c^I_{H,2}$, it simultaneously decreases $c^I_{H,1}$ to a lesser extent so that H-agents actually end up choosing the efficient allocation from the menu, i.e., the executability constraints are satisfied:

$$u\left(c_1^*\right) + \beta u\left(c_2^*\right) \geq u\left(c^I_{H,1}\right) + \beta u\left(c^I_{H,2}\right).$$

The non-sophisticated agents work efficiently based on the incorrect expectation of consuming the imaginary allocation.

Figure 1 provides an illustration of how PAM relaxes the incentive constraints. The flatter solid (blue) curve represents the indifference curve from the perspective of $t = 0$ at efficient consumption $(c_1^*, c_2^*) = (c^*_{H,1}, c^*_{H,2}) = (c^*_{L,1}, c^*_{L,2})$. The present-biased agents value retirement consumption less at $t = 1$ than at $t = 0$, so the steeper solid (red) curve represents the indifference curve from the perspective of $t = 1$ at allocation (c_1^*, c_2^*). For the efficient allocations to be implemented, the imaginary allocations have to be in the area below the red curve and above the blue curve. This would ensure the executability constraint is satisfied. Furthermore, the incentive constraints provide upper and lower

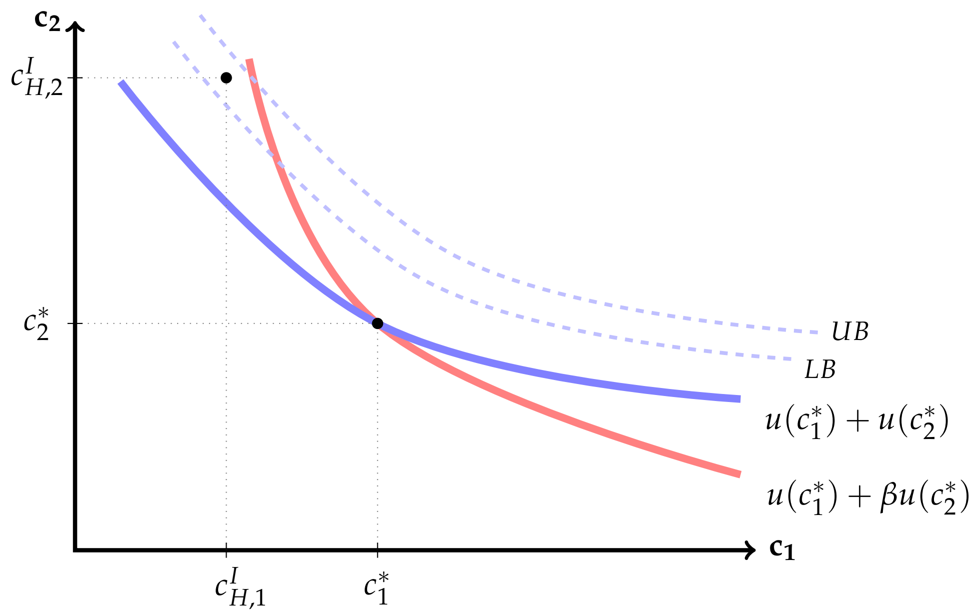

Figure 1: Finding the imaginary allocation

bounds on the imaginary allocations:

$$UB \geq \beta \left[u(c^I_{H,1}) + u(c^I_{H,2}) \right] \geq LB.$$

The upper bound is derived from the incentive constraint for L-agents (if imaginary consumption is too large then L-agents would pretend to be H-agents) and the lower bound is derived from the incentive constraint for H-agents:

$$UB = \sum_{t=0}^{1} \beta^t \left[h\left(\frac{y^*_{H,t}}{\theta_L}\right) - h\left(\frac{y^*_{L,t}}{\theta_L}\right) \right] + \beta \left[u(c^*_1) + u(c^*_2) \right]$$

and

$$LB = \sum_{t=0}^{1} \beta^t \left[h\left(\frac{y^*_{H,t}}{\theta_H}\right) - h_t\left(\frac{y^*_{L,t}}{\theta_H}\right) \right] + \beta \left[u(c^*_1) + u_2(c^*_2) \right].$$

Figure 1 shows that the imaginary consumption path has to be within the dashed lines (the lower bound and upper bound) to satisfy incentive compatibility.

When the utility from consumption is unbounded, the indifference curves are bounded away from the axis. Hence, it is always possible to find imaginary allocations that satisfy incentive compatibility, preference arbitrage and executability constraints by increasing $c^I_{H,2}$ and decreasing $c^I_{H,1}$. In other words, when u is unbounded, the government can always decrease consumption in $t = 1$ and load the information rent on retirement consumption to simultaneously satisfy both incentive compatibility and executability.

2 Two-Type Case for CCM

To see how the threat allocations within CCM relax the incentive constraints, consider an economy with fully sophisticated present-biased agents who share the same present bias β. Consider an economy with only two productivity types: $\Theta = \{\theta_L, \theta_H\}$, where $\theta_H > \theta_L$. Assume that the utility from consumption $u(c)$ is unbounded (Assumption 1 of Yu (2020)). I will show how the government can elicit the private productivity with threat allocations and implement the efficient allocation.

In $t = 0$, agents report their productivity $\theta_m \in \Theta$ and are assigned the efficient allocation $\left(c^*_0, y^*_{m,0}\right)$ for $t = 0$. In $t = 1$, agents face a menu of allocations. The threat allocation is placed within the menu C_L to deter H-agents from misreporting downwards. The notation for the threat allocation is simplified from $\left(c^T_{L|H}, y^T_{L|H}\right)$ to $\left(c^T_L, y^T_L\right)$. The menu for L-agents in a CCM is thus:

$$C_L = \left\{ \left(c^*_1, y^*_{L,1}, c^*_2\right), \left(c^T_{L,1}, y^T_{L,1}, c^T_{L,2}\right) \right\},$$

while the menu for H-agents only consists of the efficient allocation:

$$C_H = \left\{ \left(c^*_1, y^*_{H,1}, c^*_2\right) \right\}.$$

The government constructs the threat allocation such that it has higher than efficient output, $y^T_{L,1} > y^*_{L,1}$, and frontloaded consumption, $u'_1\left(c^T_{L,1}\right) < u'_2\left(c^T_{L,2}\right)$. The frontloaded consumption is tempting to the present-biased agents in $t = 1$, but only H-agents can choose it, because it is easier for them to produce the required higher than efficient output $y^T_{L,1}$. Since agents are aware of their present bias, in $t = 0$, they wish to prevent their futureselves from consuming the threat allocation, because the frontloaded consumption path exacerbates future intertemporal distortion. Therefore, H-agents would rather work efficiently in $t = 0$ to ensure higher retirement consumption, which is provided by the efficient consumption path.

Since utility from consumption is unbounded, the efficient allocation can be implemented by fully relaxing the incentive constraints through the continual decrease of $c^T_{L,2}$:

$$
\begin{aligned}
u\left(c_0^*\right) - h\left(\frac{y^*_{H,0}}{\theta_H}\right) + \beta\left[u\left(c_1^*\right) - h\left(\frac{y^*_{H,1}}{\theta_H}\right) + u\left(c_2^*\right)\right] \\
\geq u\left(c_0^*\right) - h\left(\frac{y^*_{L,0}}{\theta_H}\right) + \beta\left[u\left(c^T_{L,1}\right) - h\left(\frac{y^T_{L,1}}{\theta_H}\right) + u\left(c^T_{L,2}\right)\right].
\end{aligned}
$$

Notice that the H-agents evaluate downward misreports using the threat allocation, because the sophisticated H-agents expect to choose the threat allocation from C_L. To achieve this, the government simultaneously increases $c^T_{L,1}$ while it decreases $c^T_{L,2}$ to make it more appealing for misreporting agents, i.e., the threat constraint is satisfied:

$$
u\left(c^T_{L,1}\right) - h\left(\frac{y^T_{L,1}}{\theta_H}\right) + \beta u\left(c^T_{L,2}\right) \geq u\left(c_1^*\right) - h\left(\frac{y^*_{L,1}}{\theta_H}\right) + \beta u\left(c_2^*\right).
$$

Finally, the government needs to make sure that the actual L-agents wouldn't choose the threat allocation, so it increases $y^T_{L,1}$ to satisfy the executability constraint:

$$
u\left(c_1^*\right) - h\left(\frac{y^*_{L,1}}{\theta_L}\right) + \beta u\left(c_2^*\right) \geq u\left(c^T_{L,1}\right) - h\left(\frac{y^T_{L,1}}{\theta_L}\right) + \beta u\left(c^T_{L,2}\right).
$$

Intuitively, in this setup, agents who are aware of their bias willingly trade in the information rent for commitment.

Figures 2 and 3 illustrate how CCM works. Let $\Phi^i_{j,k} = u\left(c^i_{j,1}\right) - h\left(\frac{y^i_{j,1}}{\theta_k}\right)$ and $\Phi^i_{k,k} = \Phi^i_k$, where $i \in \{R, T\}$ and $j, k \in \{L, H\}$. In essence, $\Phi_{j,k}$ denotes the utility of a k-agent in $t = 1$ who reported productivity θ_j in $t = 0$. I will show how to construct threat allocations to deter misreporting. Then, I show how it can be adjusted so that truthful agents would never choose it. From incentive compatibility, threat and executability constraints, the efficient and threat allocations have to satisfy:

$$\Phi^T_{L,H} > \Phi^R_{L,H} > \Phi^R_L > \Phi^R_H, \text{and } c^*_2 > c^T_{L,2}.$$

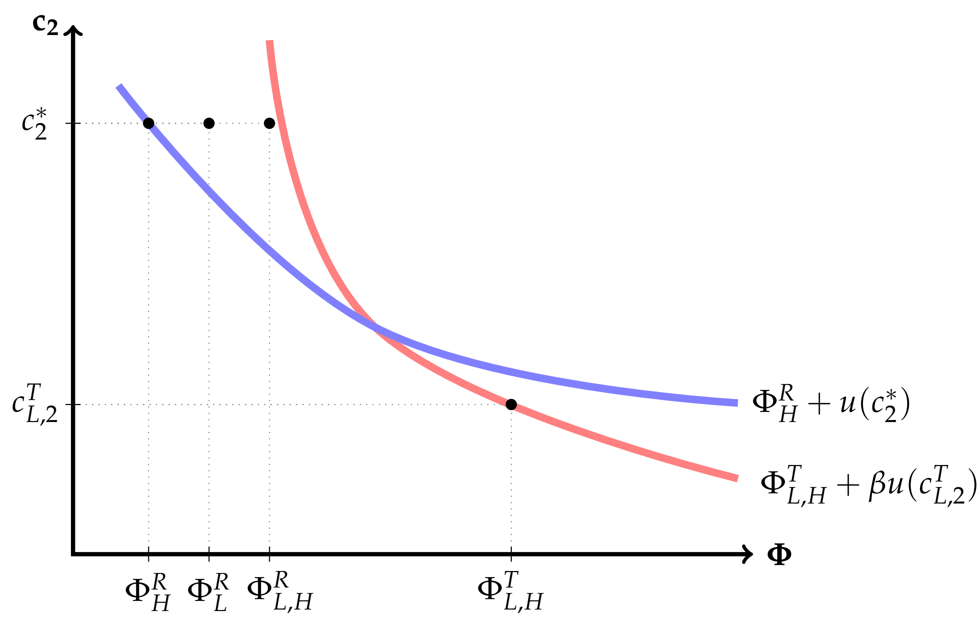

Figure 2: Finding the threat allocation: part I

Figure 2 shows how incentive compatibility restricts the set of threat allocations. The steeper solid (red) curve represents the indifference curve from the perspective of $t = 1$ for the H-agent who pretended to be θ_L. The flatter solid (blue) curve represents the indifference curve from the perspective of $t = 0$ for the H-agent who reported truthfully. The threat allocation needs to be designed such that it is below the blue curve—the incentive compatibility constraint holds. It also needs to be chosen so that the utility from consuming it in $t = 1$ is higher than the efficient allocation for L-agents—the threat constraint holds. Figure 2 shows a combination of $c^T_{L,2}$ and $\Phi^T_{L,H}$ that satisfies both incentive and threat constraints. More specifically, when utility u is unbounded, the government can choose $c^T_{L,2}$ and $\Phi^T_{L,H}$ such that the incentive compatibility and threat constraints are satisfied by decreasing $c^T_{L,2}$ and increasing $\Phi^T_{L,H}$.

Next, I show that $\left(c^T_{L,1}, y^T_{L,1}\right)$ can be chosen so that the executability constraint is satisfied. To see this, fix the choice of $c^T_{L,2}$ and $\Phi^T_{L,H}$ at the level shown in Figure 2. If the threat satisfies incentive compatibility, threat and executability constraints, it implies

$$\Delta u_2 \geq \Phi^T_{L,H} - \Phi^R_H + \frac{1}{\beta}\Delta h_0 > \Phi^T_{L,H} - \Phi^R_{L,H} \geq \beta \Delta u_2 \geq \Phi^T_L - \Phi^R_L,$$

where $\Delta u_2 \equiv \left[u(c^*_2) - u(c^T_{L,2})\right]$ and $\Delta h_0 = h\left(\frac{y^*_{H,0}}{\theta_H}\right) - h\left(\frac{y^*_{L,0}}{\theta_H}\right)$. The problem now is to find $c^T_{L,1}$ and $y^T_{L,1}$ such that

$$u_1(c^T_{L,1}) - h_1\left(\frac{y^T_{L,1}}{\theta_H}\right) = \Phi^T_{L,H}$$

and satisfies the executability constraint,

$$\beta \Delta u_2 \geq \Phi^T_L - \Phi^R_L.$$

Figure 3 shows how the government can increase $y^T_{L,1}$ to discourage L-agents from choosing the threat allocation. The flatter thick solid (blue) curve represents the indifference curve of Φ for the H-agents at allocation $\left(c^*_1, y^*_{L,1}\right)$. The steeper solid (red) curve represents the indifference curve of Φ for the L-agents at allocation $\left(c^*_1, y^*_{L,1}\right)$. The dashed (blue) curve represents the indifference curve of Φ for the H agent at allocation $\left(c^T_{L,1}, y^T_{L,1}\right)$, chosen so that

$$u_1(c^T_{L,1}) - h_1\left(\frac{y^T_{L,1}}{\theta_H}\right) = \Phi^T_{L,H},$$

where $\Phi^T_{L,H}$ was chosen in Figure 2. Since u is unbounded, it can increase $c^T_{L,1}$ so $u_1(c^T_{L,1}) -$

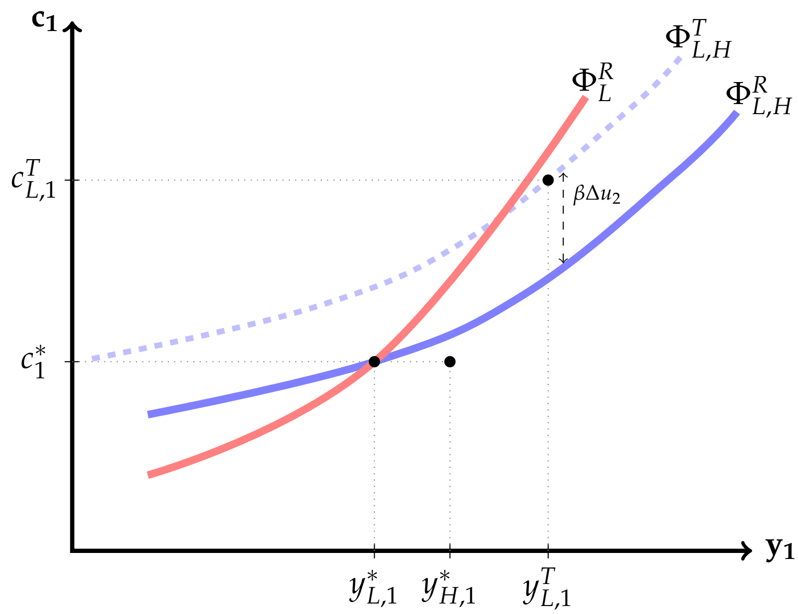

Figure 3: Finding the threat allocation: part II

$h_1\left(\frac{y^T_{L,1}}{\theta_H}\right) = \Phi^T_{L,H}$ while $y^T_{L,1}$ is increased to satisfy the executability constraint.

3 Conclusion

Yu (2020) extends the mechanisms above to a setting with private sophistication and present bias, and multiple productivity types. Yu (2020) also provides policy implementations of these mechanisms. For details on how PAM and CCM can be extended to richer settings, please see Yu (2020).

References

Yu, Pei Cheng, "Optimal Retirement Policies with Present-Biased Agents," *Journal of European Economic Association*, 2020, *forthcoming.*