Altruism and Risk Sharing in Networks

Renaud Bourlès, Yann Bramoullé & Eduardo Perez-Richet

Aix-Marseille School of Economics & Sciences Po

February 2020

Introduction

- Informal transfers are still widespread in our economies.
  - Transfers in cash or in kind which are not market transactions.
- Key stylized facts:
  - Large, even in high-income countries.
  - Interact with markets and public transfers.
  - Generate redistribution and not too inefficient insurance.
  - Flow through family and social networks.

Introduction

- Growing theoretical literature developing models of informal transfers in networks.
- Pareto-constrained risk sharing arrangements under network constraints.
  - Social collateral, Ambrus, Mobius & Szeidl (AER 2014).
  - Local information, Ambrus, Gao & Milan (WP 2017).
- Altruism in networks, Bourlès, Bramoullé & Perez-Richet (ECA 2017).
  - Altruism à la Becker, structured through a network.
  - Nash equilibria of the game of transfers, for non-stochastic incomes.

Introduction

- In this new paper, we study the risk sharing implications of altruism networks.
  - Incomes are stochastic, transfers conditional on incomes as in BBP (2017).
  - Becker (JPE 1974)'s early intuition: "The head's concern about the welfare of other members provides each, including the head, with some insurance against disasters."
  - Never studied in a network context.
- We find that altruism networks have a strong impact on risk and generate specific patterns of consumption smoothing.

Introduction: altruism in networks

- Advances the economics of altruism.
  - Initiated by Becker (JPE 1974) and Barro (JPE 1974).
  - Large literature but irrealistic structures: Small groups of completely connected agents or linear dynasties.
- However, family ties form complex networks.
  - Well-known from human genealogy.
  - Argued early on by Bernheim & Bagwell (JPE 1988) but had not been explored by economists.

Introduction: overview of the results

- Altruistic transfers achieve efficient insurance:
  - For any shock, when the network of perfect altruistic ties is strongly connected.
  - For small shocks, when the network of transfers is weakly connected.
- Informal insurance better when the altruism network has lower average path length.
  - Disproportionate impact of bridges and long-distance links.
- We uncover rich structural effects.
  - More central agents have lower consumption variance. Closer agents have more correlated consumption streams. New link can increase variance of indirect neighbors.

Model: informal transfers

- Agent i has income y_i^0 and may give $t_{ij} \geq 0$ to agent j.
  - Matrix $\mathbf{T} = (t_{ij})$ represents the network of informal transfers.
- Consumption y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_k t_{ki}$$

- Aggregate income is conserved: $\sum_i y_i = \sum_i y_i^0$.

Model: altruism in networks

- Agents care about others' well-being:

$$v_i(\mathbf{y}) = u_i(y_i) + \sum_j \alpha_{ij} u_j(y_j)$$

- Coefficient $\alpha_{ij} \in [0, 1]$ measures the strength of the altruistic link from i to j.
  - Network of altruism (α_{ij}) describing the structure of social preferences.
- i may care about j but not about j's friends. Interests of a giver and a receiver may be misaligned.

Model: altruism in networks

- Noncooperative game: Agents makes transfers to maximize their altruistic utilities.
  - Transfers by an agent depend on transfers made by others.
- $\mathbf{T}$ is a Nash equilibrium iff (1) $t_{ij} > 0 \Rightarrow u_i'(y_i) = \alpha_{ij} u_j'(y_j)$ and (2) $\forall i, j, u_i'(y_i) \geq \alpha_{ij} u_j'(y_j)$.
  - If $u_i(y) = -e^{-Ay}$, (1) $t_{ij} > 0 \Rightarrow y_i = y_j + (-\ln(\alpha_{ij}))/A$ and (2) $\forall i, j, y_i \leq y_j + (-\ln(\alpha_{ij}))/A$.
- An agent does not let the consumption of a poorer friend become too much lower than his own.

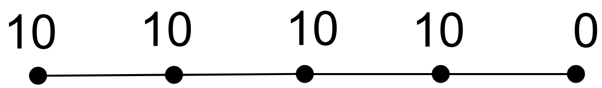
Model: altruism in networks

Proposition (BBP 2017) *A Nash equilibrium always exists. Equilibrium consumption* $\mathbf{y}$ *is unique. Generically in* $\boldsymbol{\alpha}$, *there is a unique Nash equilibrium* $\mathbf{T}$ *and it has a forest structure.*

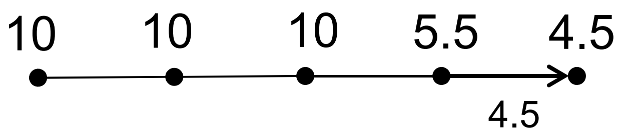
- Emergence of transfer intermediaries.
  - Give to poorer friends part of the money received from richer friends.
  - Shocks propagate in the altruism network.

Equilibria on the line

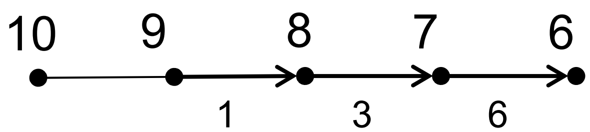
u CARA, links have same strength: $-\ln(\alpha)/A=1$



One round of
best response



The unique Nash
equilibrium

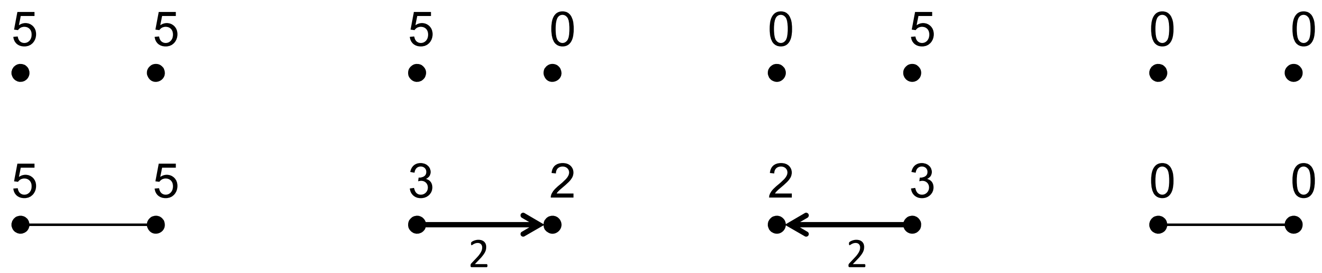


Altruism and risk

- Suppose now that incomes are stochastic.
  - Example with 2 agents, common CARA, $\alpha_{12} = \alpha_{21} = \alpha$.
  - *Altruistic transfers mimick a classical insurance scheme*: Gives when rich, receives when poor.
  - Insurance improves as α increases.
- What happens on complex networks?

Altruistic transfers help smooth consumption

u CARA with -ln(α)/A=1 and iid binary incomes



Model: efficient insurance

Definition *Informal transfers yield efficient insurance if* $\exists \boldsymbol{\lambda} \gneq \mathbf{0}$ *such that consumption* $\mathbf{y}$ *solves*

$$\max_{\sum_i y_i = \sum_i y_i^0} \sum_i \lambda_i \mathbb{E} u_i(y_i)$$

- Classical notion underlying empirical analysis following Townsend (1994).
  - Ex-ante Pareto frontier with respect to private utilities.
- $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$ in every state of the world.
  - Common utilities and equal weights lead to equal sharing $y_i = \bar{y}^0$.

Perfect altruism

- Altruism is perfect if $\alpha_{ij} = 1$.
  - Network of perfect altruism strongly connected if any two agents indirectly connected through perfect altruistic ties.

Proposition *Informal transfers generate efficient insurance for any stochastic incomes if and only if the network of perfect altruism is strongly connected. In this case, equal Pareto weights.*

- Perfect altruism between pairs aggregate up into efficient insurance.
  - Even when the network is sparse and agents' interests are misaligned.

Imperfect altruism

- How far does society get from efficient insurance when altruism is imperfect?
  - Following Ambrus, Mobius & Szeidl, introduce distance from equal income sharing: $DISP(\mathbf{y}) = \frac{1}{n}\sum_i |y_i - \bar{y}^0|$.
- Define $c_{ij} = -\ln(\alpha_{ij})$ virtual cost and $\hat{c}_{ij}$ = least-cost of paths connecting i to j.
  - If $\alpha_{ij} \in \{0, \alpha\}$, $\hat{c}_{ij} = cd_{ij}$ where d_{ij} = network distance between i and j.

Imperfect altruism

Proposition *Assume that agents have common CARA utilities. If the altruism network is strongly connected, then*

$$DISP(\mathbf{y}) \leq \frac{1}{An^2} \sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$$

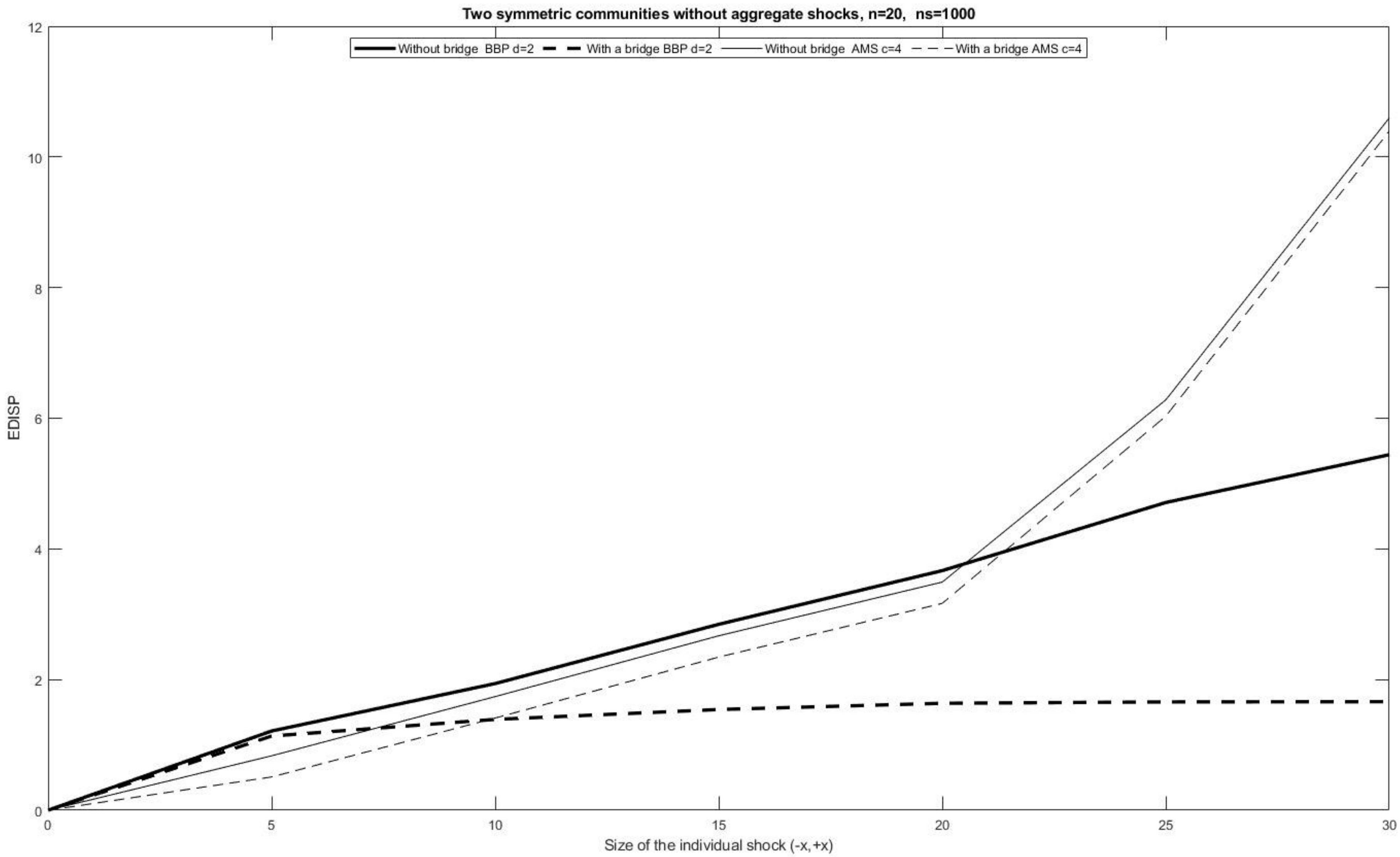
If the altruism network is not strongly connected, $\mathbb{E}DISP$ *can take arbitrarily large values.*

- Extension of average path length. If $\alpha_{ij} \in \{0, \alpha\}$,

$$\frac{1}{n^2} \sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji}) = \frac{n(n-1)}{n^2} \bar{d}$$

Large shocks: imperfect altruism

- One bridge between disconnected communities has a major impact.
  - Distance to equal sharing from arbitrarily large to bounded.
  - Contrasts to Ambrus, Mobius & Szeidl (AER 2014): capacity constraint of the bridge quickly saturated.
- Simulations: u CARA, 2 complete networks of 20 agents, $\mathbb{E}y_i^0 = 30$.
  - Idiosyncratic iid schock: $-x$ (0.5) / $+x$ (0.5).
  - We compute $\mathbb{E}DISP$ from 1000 runs.


Two symmetric communities without aggregate shocks, n=20, ns=1000
Without bridge BBP d=2
With a bridge BBP d=2
Without bridge AMS c=4
With a bridge AMS c=4
EDISP
Size of the individual shock (-x,+x)
0
2
4
6
8
10
12
0
5
10
15
20
25
30

Imperfect altruism

- Informal insurance generated by altruistic transfers subject to *small-world effects.*
  - A few-long distance connections have a disproportionate impact on overall insurance.
- Result extends to other measures of distance ($SDISP$) and utility functions (CRRA, quadratic).

Small shocks

- We next fully characterize what happens for small shocks.
- From equilibrium $\mathbf{T}$, define the *graph of transfers* $\mathbf{G}$ as $g_{ij} = 1$ if $t_{ij} > 0$ and $g_{ij} = 0$ otherwise.
  - Generically in $\boldsymbol{\alpha}$ and $\mathbf{y}^0$, small shocks $\mathbf{y}^0 + \boldsymbol{\varepsilon}$ yield the same graph of transfers.
- If i and j belong to the same weak component of $\mathbf{G}$, define $\bar{c}_{ij} = \sum_{t_{i_s i_{s+1}} > 0} c_{i_s i_{s+1}} - \sum_{t_{i_{s+1} i_s} > 0} c_{i_{s+1} i_s}$ in path from i to j.
  - Depends on transfers and their directions, can be negative.

Small shocks: main result

Theorem

(1) Suppose that for any income realization there is a Nash equilibrium with forest transfer graph $\mathbf{G}$*. Then, altruistic transfers generate efficient insurance within weak components of* $\mathbf{G}$*. If* i *belongs to weak component* C *of size* n_C*, his Pareto weight* λ_i *is such that* $\ln(\lambda_i) = \frac{1}{n_C}\sum_{j \in C} \bar{c}_{ij}$.

(2) Consider an income distribution whose support's interior is non-empty. Generically in $\boldsymbol{\alpha}$*, if society is partitioned in communities and altruistic transfers generate efficient insurance within communities, then the graph of transfers is constant across income realizations in the support's interior and these communities are equal to the weak components of the transfer graph.*

Small shocks: proof

- Idea of the proof of (1):
  - Nash: $t_{ij} > 0 \Rightarrow u_i'(y_i)/u_j'(y_j) = \alpha_{ij}$.
  - Planner: for any i, j, $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$.
  - When **G** is fixed, we find λ's such that Nash conditions equivalent to planner's conditions within a weak component.

Small shocks: efficient insurance

- Weak components of $\mathbf{G}$ constitute endogenous risk sharing communities.
- Within these components, equilibrium behavior equivalent to a planner's program.
  - Quality of insurance depends on the connectedness of $\mathbf{G}$.
- *Efficient insurance if* $\mathbf{G}$ *is weakly connected*.
  - Formalizes and extends Becker's intuition.
  - Holds with a household head or with a rich benefactor in a connected community.

Small shocks: efficient insurance

- No transfers and no insurance if **G** is empty.
  - Happens with small income differencs $|y_i^0 - y_j^0| < \varepsilon$.
  - Agents bear all the risk associated with small shocks.
  - Contrasts with social collateral: small shocks always perfectly insured.
- A small increase in α_{ij} leads to an increase in λ for j and his indirect neighbors and to a decrease for i and his indirect neighbors.

Network structure and insurance

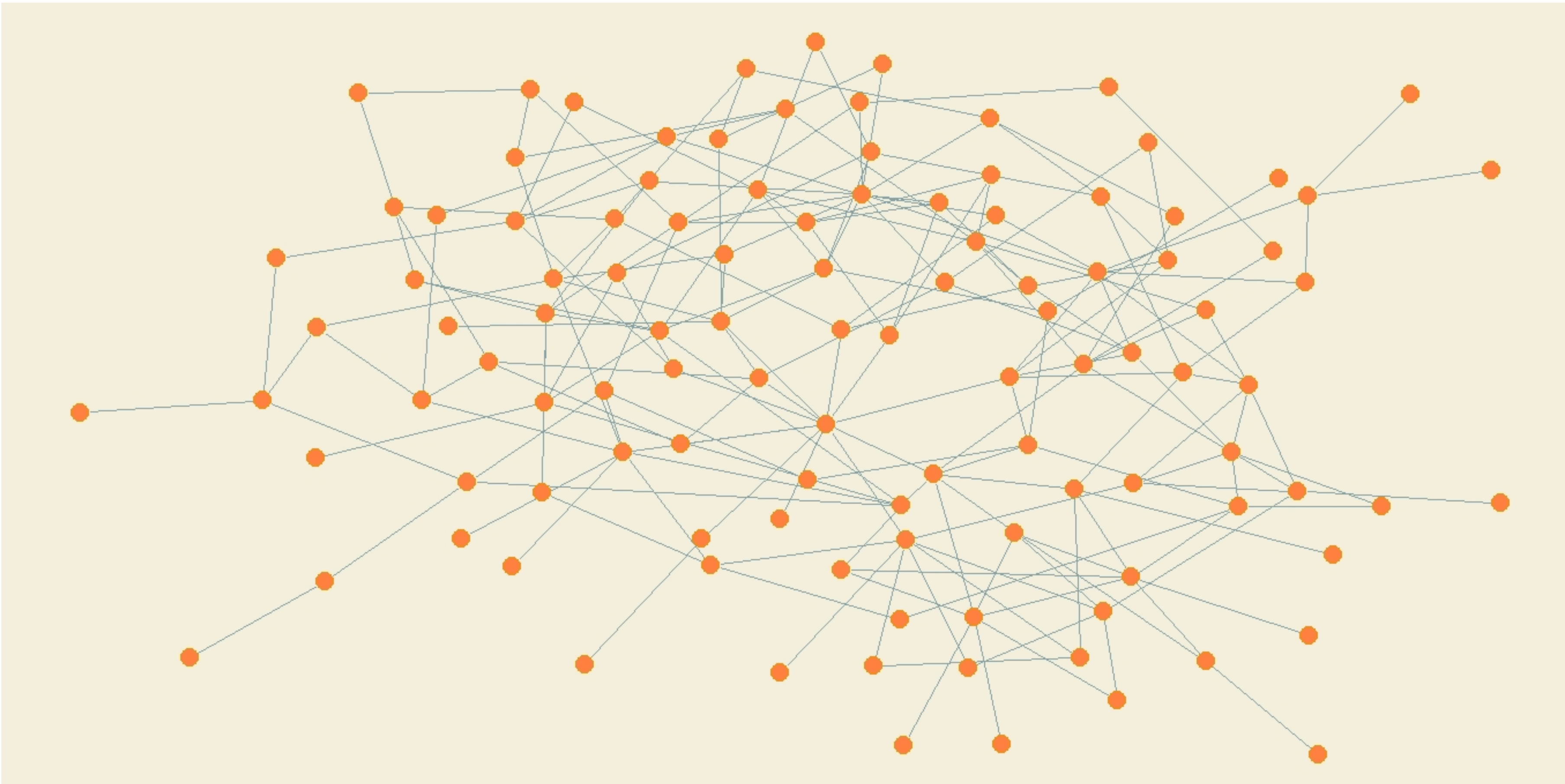
- How does informal insurance depend on the network structure?
  - Are central agents better insured?
  - Does a new link help? How does it affect indirect neighbors?
  - Correlations in consumption streams?

Network structure and insurance

- We explore these effects through numerical simulations.
  - We identify a benchmark to focus on risk-sharing.

Proposition *Under CARA, symmetric incomes and undirected ties* $\alpha_{ij} = \alpha_{ji}$, $\forall i, \mathbb{E}y_i = \mathbb{E}y_i^0$.

- CARA with $-\ln(\alpha_{ij})/A = 3$, iid symmetric binary $\{0, 20\}$.
  - Network of informal lending and borrowing from Banerjee, Chandrasekhar, Duflo & Jackson (S 2013), $n = 111$.
  - 1000 runs to recover the consumption distribution.
  - Relatively fast algorithm based on the potential property.



Network structure and insurance: centrality

- Result 1: Negative correlation between consumption variance and centrality.
  - As with social collateral (Ambrus, Mobius & Szeidl 2014), unlike under local info constraints (Ambrus, Gao & Milan 2017).

Correlation between Centralities and Consumption Variance

	Variance
Degree	-0.7612***
Between	-0.5094***
Eigen	-0.6741***

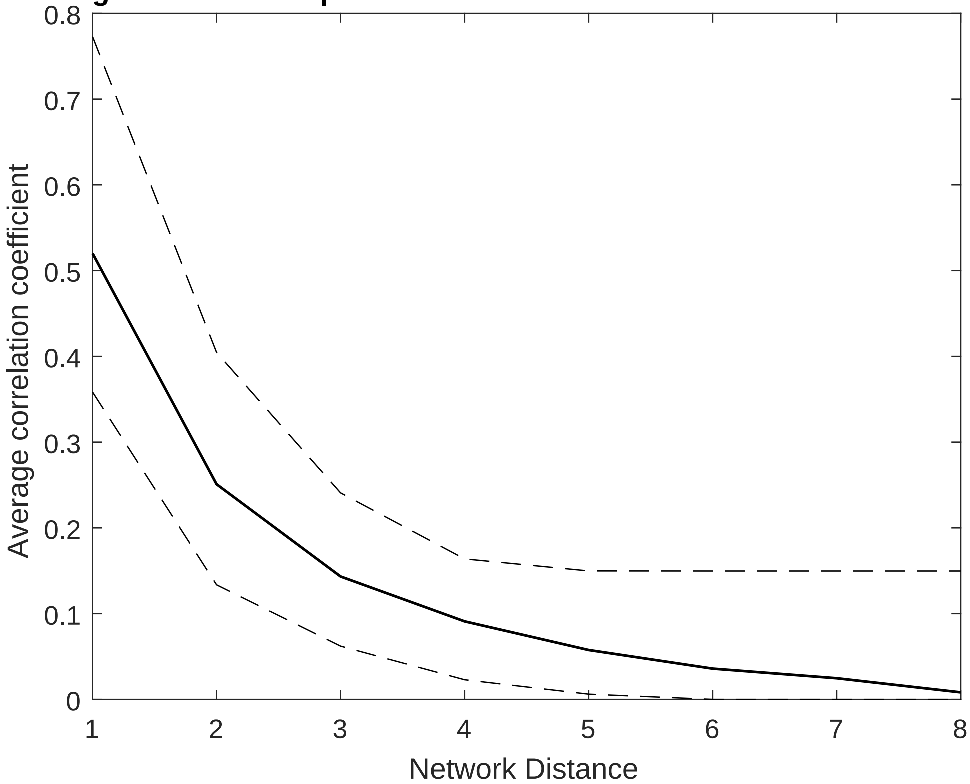
***denotes statistical significance at the 1% level

Network structure and insurance: consumption correlation

Proposition *If incomes are independent, then* $\forall i, j, cov(y_i, y_j) \geq 0$

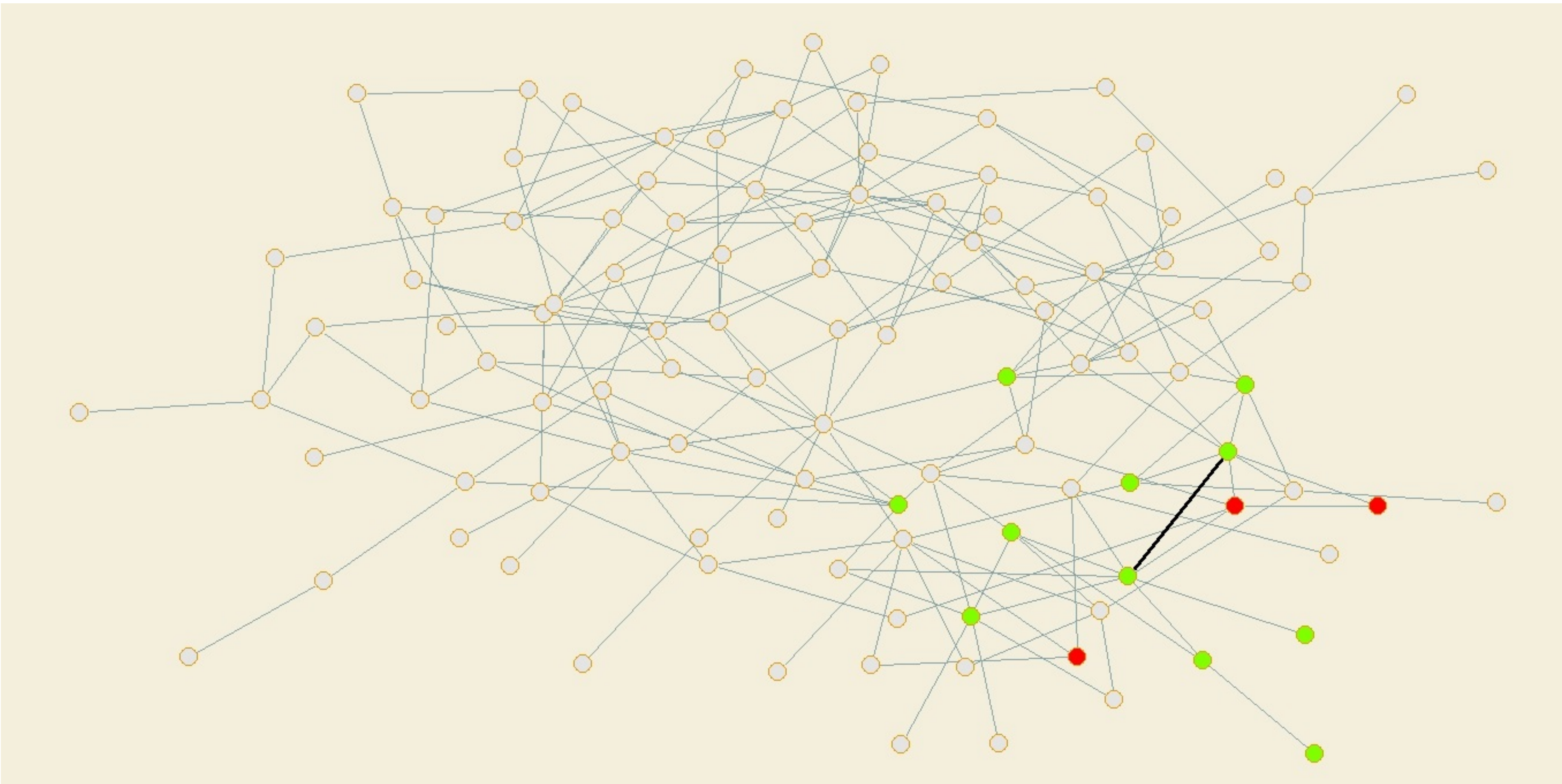
- Altruistic transfers induce correlation between consumption streams across agents.
- Result 2: Positive correlation in consumption decreases with distance in the network.

Correlogram of consumption correlations as a function of network distance



Network structure and insurance: new link

- Result 3: New link reduces the consumption variance of both agents.
- Result 4: New link can increase or decrease the consumption variance of indirect neighbors.
  - New source of indirect support vs competitor for neighbors' help.



Conclusion

- We analyze the risk sharing properties of altruism networks.
- Altruistic transfers generate efficient insurance:
  - When the network of perfect altruism is strongly connected.
  - When the graph of transfers is invariant and weakly connected.
- Informal insurance tends to be better:
  - When the average path length of the altruism network is lower.
  - On small shocks, when the network of transfers is more connected.
- Rich structural effects.